

Code No: A10407

# MLR INSTITUTE OF TECHNOLOGY

(An Autonomous Institution)

II B.Tech I Semester Regular Examinations- December-2016

## RANDOM VARIABLES AND STOCHASTIC PROCESSES

(ECE)

Time: 3 hours

Max. Marks: 75

- Note: 1. This question paper contains two parts A and B  
 2. Part A is compulsory which carries 25 marks .Answer all Questions in part A.  
 3. Part B consists of 5 units. Answer any one full question from each unit. Each question carries 10 Marks and may have a, b,c sub questions.

### PART-A

- 1 a) what are the conditions for a function to be random variable? [2M]
- b) Define Rayleigh density and distribution function. [2M]
- c) State Central Limit theorem. [2M]
- d) Distinguish between stationary and non-stationary random processes. [2M]
- e) Define Power density Spectrum. [2M]
- 2 a) what is Poisson random variable .Explain in brief? [3M]
- b) Explain the concept of Transformations of random variable . [3M]
- c) State and prove the properties of joint characteristics function. [3M]
- d) Explain briefly about time average and ergodicity. [3M]
- e) Write a short notes on the cross Power spectral density? [3M]

### PART-B

- 3 (a) Distinguish between distribution function and density function .Explain their properties.[5M]
- (b) A random variable has a pdf  $f_X(x) = \frac{C}{\sqrt{25-x^2}} - 5 < x < 5$ . Find [5M]
  - (i) The value of C
  - (ii)  $P(X > 2)$
  - (iii)  $P(X < 3)$
  - (iv)  $P(X < 3/X > 2)$ .

### OR

- 4 (a) Define Exponential density and distribution function. Explain with their plots. [5M]
- (b) The Probabilities of a random variable X are given as when  $x_1 = \frac{1}{2}$  ,  $P(x_1) = \frac{1}{2}$  [5M]
 

and  $x_2 = \frac{-1}{2}$  ,  $P(x_2) = \frac{1}{2}$ . Find the Moment Generating Function and the first four Moments.
- 5 (a) state and prove the properties of variance of a random variable. [5M]
- (b) Find the density function of the random variable X if the characteristic function [5M]

$$\text{is } \phi_X(\omega) = \begin{cases} 1 - |\omega|, & |\omega| \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

### OR

- 6 (a) Explain in detail about the central Moments. [5M]
- (b) A random variable X has  $\bar{X} = -3, \overline{X^2} = 11$ , and  $\sigma_X^2 = 2$ . For a new random variable  $Y = 2X - 3$ , Find (a)  $\bar{Y}$  (b)  $\overline{Y^2}$  (c)  $\sigma_Y^2$  [5M]

7 (a) Define and explain the conditional distribution and density function of two variables  $X$  and  $Y$ . [5M]

(b) The joint CDF of a random variable  $X$  and  $Y$  is given by [5M]

$$F_{X,Y}(x, y) = \begin{cases} (1 - e^{-ax})(1 - e^{-bx}), & x \geq 0, y \geq 0, a, b > 0 \\ 0, & \text{elsewhere} \end{cases}$$

(i) Find the marginal CDFs of  $X$  and  $Y$

(ii) Show that  $X$  and  $Y$  are independent

(iii) Find  $P(X \leq 1, Y \leq 1), P(X \leq 1), P(Y \geq 1),$  and  $P(X > x, Y > y)$

**OR**

8 (a) Explain the Gaussian density function for  $N$  random variables. [5M]

(b) the joint density function of  $X$  and  $Y$  is

$$f_{X,Y}(x, y) = \begin{cases} \frac{1}{100}, & 0 < x < 5, 0 < y < 20 \\ 0, & \text{elsewhere} \end{cases}$$

Find the expected value of the function.

i)  $XY$     ii)  $X^2Y$     iii)  $(XY)^2$

9 (a) Explain how random processes are classified with neat sketches. [5M]

(b) A stationary process has an autocorrelation function given by

$$R_{XX}(\tau) = \frac{25\tau^2 + 36}{6.25\tau^2 + 4}$$

Find the mean value, mean square value and variance of the process. [5M]

**OR**

10 (a) Write short notes on Gaussian random processes. [5M]

(b) A random process  $Y(t)$  is given by  $Y(t) = X(t)\cos(\omega t + \theta)$ , where  $X(t)$  is a [5M]

Wide sense stationary random process,  $\omega$  is a constant and  $\theta$  is a random phase

independent of  $X(t)$ , uniformly distributed on  $(-\pi, \pi)$ . Find (i)  $E[Y(t)]$  (ii)  $R_{YY}(\tau)$ .

11 (a) State and prove Wiener –Khinchin relations. [5M]

(b) The spectral density of a wide sense stationary random process  $X(t)$  is given by

$$S_{XX}(\omega) = \frac{\omega^2}{\omega^4 + 13\omega^2 + 36}. \text{ Find the autocorrelation and average power of the process. [5M]}$$

**OR**

12 (a) write short notes on cross power density spectrum? [5M]

(b) A random process  $X(t)$  has the autocorrelation function  $R_{XX}(\tau) = \frac{A_0^2}{2} \sin(\omega_0 \tau)$

Find the power spectral density  $S_{XX}(\omega)$ . [5M]