

# MLR INSTITUTE OF TECHNOLOGY

(Autonomous Institute)

I B.Tech I Sem Supplementary/Improvement Examinations, February-2016

## MATHEMATICS-I

(Common to All)

- Note: 1. This question paper contains two parts A and B.  
 2. Part A is compulsory which carries 25 marks. Answer all Questions in part A.  
 3. Part B consists of 5 units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

### PART A

(25 Marks)

1. a) Show that the differential equation  $x dy + y dx = \frac{y dx - x dy}{y^2}$  is exact. [2M]  
 b) Write the particular integral of  $(D^2 - 2D + 1)y = e^x$ . [2M]  
 c) Verify Roll's theorem for  $f(x) = x^3 - 12x$  in  $[0, 2\sqrt{3}]$ . [2M]  
 d) Show that  $\int_0^{\infty} e^{-x^2} dx = \sqrt{\frac{\pi}{2}}$  [2M]  
 e) Find Laplace transform of  $e^{-3t}(2 \cos 5t - 3 \sin 5t)$  [2M]
2. a) Obtain differential equation of all circles which passes through the origin whose centres lie on x-axis [3M]  
 b) Solve  $(D^4 + a^4)y = 0$ . [3M]  
 c) Expand  $f(x) = e^x$  in Taylor's series about  $x=1$ . [3M]  
 d) Evaluate  $\int_0^2 \int_0^3 xy dx dy$  [3M]  
 e) Find  $L^{-1} \left[ \frac{1}{(s-a)(s-b)} \right]$  using convolution [3M]

### PART B

[50Marks]

3. a) Solve  $(1 + xy) y dx + (1 - xy) x dy = 0$  [5M]  
 b) A body is heated to  $110^\circ\text{C}$  and placed in air at  $10^\circ\text{C}$ . After 1 hour, its temperature is  $60^\circ\text{C}$ . How much additional time is required for it to cool to  $30^\circ\text{C}$ ? [5M]

OR

4.a) Show that the family of parabolas  $x^2 = 4a(y+a)$  is self orthogonal. [5M]

b) If 30% of radio active substance disappears in 10 days how long will it take for 90% of it to disappear? [5M]

5. a) Solve  $(D^2+5D-6)y = \sin 4x \sin x$  [5M]

b) Solve  $(D^2 - 6D + 13)y = 8e^{3x} \sin 2x$  [5M]

OR

6.a) Using the method of variation of parameters solve  $(D^2-3D+2)y = \frac{1}{1+e^{-x}}$  [5M]

b) A particle is executing S.H.M with amplitude 5 meters and time 4 seconds. Find the required by the particle in passing between the points which are distance 4 and 2 meters from the centre of force and on the same side of it. [5M]

7 a) Show that  $\frac{h}{1+h^2} < \tan^{-1}h < h$  when  $h > 0$  using Lagrange's mean value theorem [5M]

b) Test the function  $x^4+y^4-x^2-y^2+1$  for maxima, minima. [5M]

OR

8 a) Show that  $u = \frac{x}{y}$ ,  $v = \frac{x+y}{x-y}$  are functionally dependent and find the relation. [5M]

b) Show that the rectangle parallelepiped of maximum volume that can be inscribed in a sphere is a cube. [5M]

9. Change the order of integration and evaluate  $\int_0^1 \int_{x^2}^{2-x} xy \, dx \, dy$  [10M]

OR

10.a) Evaluate  $\int_0^{\pi/2} \sin^{11} \theta \, d\theta$  using Beta and Gamma function. [5M]

b) Evaluate  $\int_0^1 \left(\log \frac{1}{x}\right) dx$  in terms of Gamma function. [5M]

11.a) Find  $L\left[\int_0^t \frac{e^{-4t} \sin 3t \, dt}{t}\right]$  [5M]

b) Find  $L^{-1}\left[\cot^{-1}\left(\frac{s+a}{b}\right)\right]$  [5M]

OR

12. Solve  $y''' - 3y'' + 3y' - y = t^2 e^t$  given  $y(0)=1, y'(0)=0, y''(0)=-2$  using Laplace transforms. [10M]

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