



### Instructions:

1. Write your Roll Number on every page of this booklet.
2. Fill out the attached performance card. **Do not detach it from this booklet.**
3. Booklet consists of 26 pages (excluding this sheet) and 7 questions.
4. Questions consist of sub-questions. Write your **detailed answer** in the **blank space** provided below the sub-question and **final answer** to the sub-question in the **smaller box** which follows the blank space.
5. Extra sheets are also attached at the end in case you need more space. You may also use these extra sheets for rough work.
6. Computational tools such as calculators, mobiles, pagers, smart watches, slide rules, log tables etc. are **not** allowed.
7. **This entire booklet must be returned.**

### Table of Information

Speed of light in vacuum	$c = 3.00 \times 10^8 \text{ m}\cdot\text{s}^{-1}$
Planck's constant	$h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s}$
Universal constant of Gravitation	$G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2\cdot\text{kg}^{-2}$
Magnitude of the electron charge	$e = 1.60 \times 10^{-19} \text{ C}$
Mass of the electron	$m_e = 9.11 \times 10^{-31} \text{ kg} = 0.51 \text{ MeV}\cdot c^{-2}$
Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-8} \text{ W}\cdot\text{m}^{-2} \cdot \text{K}^{-4}$
Permittivity constant	$\epsilon_0 = 8.85 \times 10^{-12} \text{ F}\cdot\text{m}^{-1}$
Value of $1/4\pi\epsilon_0$	$= 9.00 \times 10^9 \text{ N}\cdot\text{m}^2\cdot\text{C}^{-2}$
Permeability constant	$\mu_0 = 4\pi \times 10^{-7} \text{ H}\cdot\text{m}^{-1}$
Universal Gas Constant	$R = 8.31 \text{ J}\cdot\text{K}^{-1}\cdot\text{mole}^{-1}$
Molar mass of air	$= 29.0 \text{ kg}\cdot\text{kmol}^{-1}$

1. A parallel plate capacitor with plate area  $A$  and separation  $d$  ( $d \ll$  lateral dimensions) is filled with a dielectric of dielectric constant  $k = e^{\alpha x}$  where  $\alpha$  is a constant ( $\alpha > 0$ ) and plate is at  $x = 0$ . The capacitor is charged to a potential  $V$ . [Marks: 5]
- (a) Obtain the capacitance  $C$ . [2]

$C =$
-------

- (b) Sketch  $C$  vs  $d$ . [1]

(c) Obtain the charge on the plates.

[1]

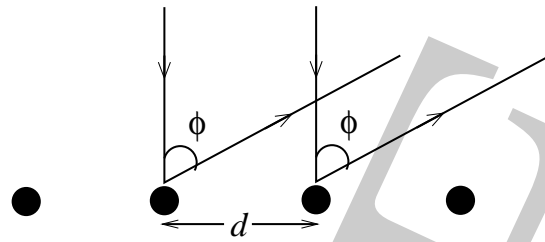
Charge =

(d) Obtain the electric field  $\vec{E}(x)$ .

[1]

$\vec{E}(x) =$

2. The figure below depicts the reflection of normally incident monochromatic wave by two neighbouring surface atoms of crystal. Angle of reflection and nearest neighbour distance are  $\phi$  and  $d$  respectively. [Marks: 5]



Two reflected waves will interfere. The interference is due to deBroglie wave of the electron accelerated from rest by less than 1 keV.

- (a) Obtain a relationship between  $\phi$ ,  $d$  and the kinetic energy ( $K$ ) of the electron with rest mass  $m_0$  for the case of constructive interference. [3]

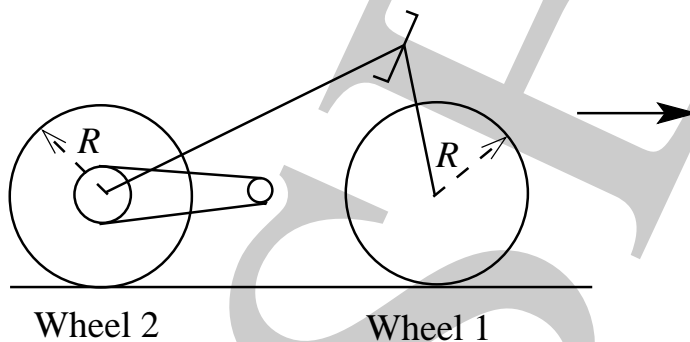
$\phi =$

- (b) Calculate  $d$  if first maxima occurs at  $\phi = 30$  deg for  $K=100$  eV electron. [2]

$d =$

3. Consider a bicycle in vertical position accelerating forward without slipping on a straight horizontal road. The combined mass of the bicycle and the rider is  $M$  and the magnitude of the accelerating torque applied to the rear wheel by the pedal and gear system is  $\tau$ . The radius and the moment of inertia of each wheel is  $R$  and  $I$  (with respect to axis) respectively. The acceleration due to gravity is  $g$ . [Marks: 8]

(a) Draw the free body diagram of the system (bicycle and rider). [2]



(b) Obtain the acceleration  $a$  in terms of the above mentioned quantities. [2]

$a =$

- (c) For simplicity assume that the centre of mass of the system is at height  $R$  from the ground and equidistant at  $2R$  from the center of each of the two wheels. Let  $\mu$  be the coefficient of friction (both static and dynamic) between the wheels and the ground. Consider  $M \gg I/R^2$  and no slipping. Obtain the conditions for the maximum acceleration  $a_m$  of the bike. [3]

$a_m =$
---------

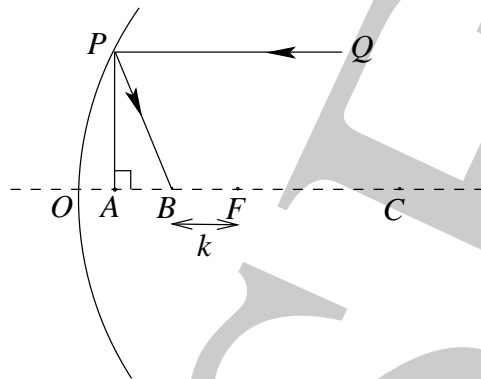
(d) For  $\mu = 1.0$  calculate  $a_m$ . [1]

$a_m =$
---------



4. The figure below depicts a concave mirror with center of curvature  $C$  focus  $F$ , and a horizontally drawn  $OFC$  as the optic axis. The radius of curvature is  $R$  ( $OC = R$  and  $OF = R/2$ ). A ray of light  $QP$ , parallel to the optical axis and at a perpendicular distance  $w$  ( $w \leq R/2$ ) from it, is incident on the mirror at  $P$ . It is reflected to the point  $B$  on the optical axis, such that  $BF = k$ . Here  $k$  is a measure of lateral aberration.

[Marks: 8]



- (a) Express  $k$  in terms of  $\{w, R\}$ .

[3]

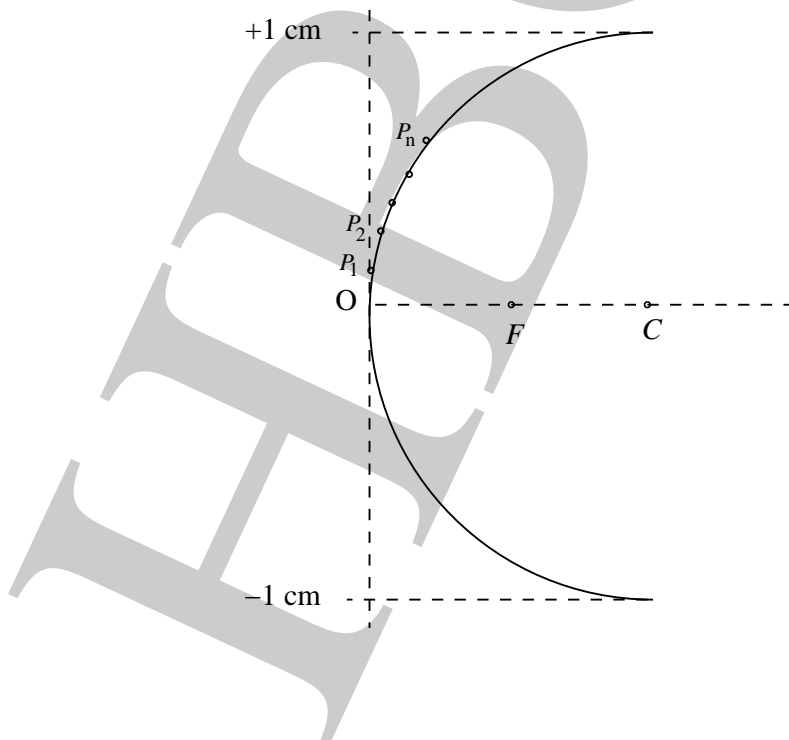
$k =$

(b) Sketch  $k$  vs  $w$  for  $w \in [0, R/2]$ .

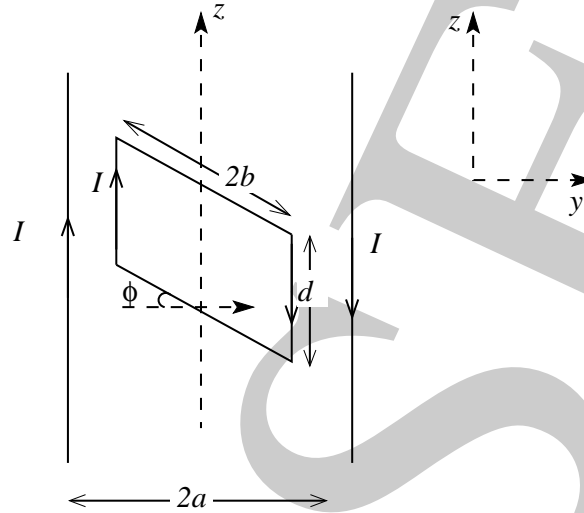
[2]

- (c) Consider points  $P_1, P_2, \dots, P_n$  on the concave mirror which are increasingly further away from the optic center  $O$  and approximately equidistant from each other (see figure below). Rays parallel to the optic axis are incident at  $P_1, P_2, \dots, P_n$  and reflected to points on the optic axis. Consider the points where these rays reflected from  $P_n, P_{n-1}, \dots, P_2$  intersect the rays reflected from  $P_{n-1}, P_{n-2}, \dots, P_1$  respectively. Qualitatively sketch the locus of these points in figure below for a mirror (shown with solid line) with radius of curvature 2 cm.

[3]



5. Two long parallel wires in the  $yz$  plane at a distance  $2a$  apart carry a steady current  $I$  in opposite directions. Midway between the wires is a rectangular loop of wire  $2b \times d$  carrying current  $I$  as shown in the figure ( $b < a < 2b$ ). The loop is free to rotate about the  $z$  axis and the currents remain fixed to  $I$  irrespective of the relative motion between the loop and the wire. [Marks: 8]



- (a) Obtain the torque tending to rotate the rectangular loop about its axis as a function of  $\phi$ . Here  $\phi$  is the angle that plane of loop makes with the plane of wires. [5]

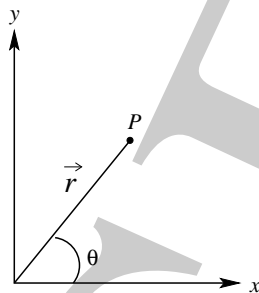
Torque is

(b) Obtain the value of  $\phi$  for which the torque is maximum?

[3]

$\phi =$

6. A planet of mass  $m$  is orbiting a star of mass  $M$  ( $M \gg m$ ) under the influence of a central force. A central force ( $F(r)\hat{r}$ ) is a force that is directed from the particle towards (or away from) a fixed point in space, the center, and whose magnitude only depends on the distance from the planet to the center. Thus gravitational force is central force and it is generally known that the orbit is a conic section of a body under the influence of gravitational force. The planet's position at some point  $P$  can be shown in polar coordinates  $(r, \theta)$  as shown in figure below:



The planet's position ( $\vec{r}$ ), velocity ( $\vec{v}$ ) and acceleration ( $\vec{a}$ ) can be written as

$$\vec{r} = r\hat{r}, \quad \vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} \quad \text{and} \quad \vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$

Consider central force to be gravitational force for parts (c-h).

[Marks: 13]

- (a) One can write scalar form of Newton's second law as

$$\frac{d^2r}{dt^2} = g(r, \theta, \dot{r}, \dot{\theta})$$

Obtain  $g(r, \theta, \dot{r}, \dot{\theta})$ .

[1]

$g(r, \theta, \dot{r}, \dot{\theta}) =$

- (b) Show that the magnitude of the angular momentum ( $L$ ) of planet is constant. [1/2]

- (c) Obtain total energy ( $E$ ) of the planet at distance  $r$  from the center of the force. Take potential energy to be zero at large  $r$ . [1]

$E =$

- (d) State the energy part  $V(r) = E - mr^2/2$  in terms of  $L$  and other quantities. [2]

$V(r) =$

(e) Sketch  $V(r)$ .

[1½]

(f) Comment on the allowed regions and shape of the orbit if planet's energy  $E > 0$ , and  $E < 0$ .

[2]

(g) Obtain the radius ( $r_0$ ) of the circular orbit in terms of  $L, m$  and related quantities.

[1]

$r_0 =$

- (h) Hence forward we take that orbit is circular of radius  $r_0$  and planet is slightly disturbed from its position such that its position is  $r = r_0 + \delta$  where  $\delta/r_0 \ll 1$ . Show that planet will oscillate simple harmonically around mean position  $r_0$ . Obtain time period ( $T_r$ ) for these radial oscillations. [2]

$T(r) =$

- (i) For this part we assume that planet moves under the force expressible as a power of  $r$  as  $F = -cr^n$  where  $c$  ( $c > 0$ ) is a constant. For what values of  $n$  a stable orbit is possible? [2]



$n =$

7. **Brunt-Vaisälä Oscillations:** A balloon containing inert gas Helium is used to study the weather. It rises up and rests at an equilibrium height in the atmosphere, where its weight is exactly balanced by the upward buoyant force. We shall assume ideal gas law to hold for all processes and neglect the mass of the balloon. If the balloon is displaced vertically, it is often found to oscillate about the equilibrium position. The frequency of this oscillation is called “Brunt-Vaisälä” frequency. We shall investigate this phenomenon. [Marks: 13]

- (a) Suppose the balloon is displaced upwards adiabatically from the equilibrium position. In the displaced position, let  $T_b$  and  $m_b$  be the temperature and molar mass of the gas in the balloon. Let  $T_a$  and  $m_a$  be the temperature and molar mass of the outside atmosphere at the same level. Let  $dT_b/dz$  and  $dT_a/dz$  be the rate of change of temperature with height for the gas in the balloon and the outside air respectively where the positive  $z$  axis is vertically upwards and  $z$  is the height from the ground.

Derive and Calculate the lapse rate ( $\Gamma_a = dT_a/dz$ ) for atmosphere given that the molar mass of air is  $29 \text{ kg}\cdot\text{kmol}^{-1}$  and the gas constant is  $R = 8.31 \text{ J}\cdot\text{K}\cdot\text{mol}^{-1}$ . Assume that acceleration due to gravity ( $g$ ) remains constant and all air processes are adiabatic. [1]

$\Gamma_a =$
--------------

- (b) Assuming that the pressure inside the balloon is the same as that outside, derive an expression for  $\Gamma_b = dT_b/dz$  in terms of the molar specific heat at constant pressure ( $C_b$ ) of the gas in the balloon. [2]

$\Gamma_b =$

- (c) Obtain an expression for the vertical acceleration ( $\ddot{z}$ ) of the balloon in terms of temperatures and molar masses. [2]

$\ddot{z} =$

- (d) Find the equilibrium height ( $z_0$ ) in terms of molar masses,  $C_b$  and relevant quantities? Assume that for both ballon and air,  $T = T_0$  at  $z = 0$ . [2]

$z_0 =$

- (e) State the condition for balloon to oscillate simple harmonically. Also obtain the frequency of oscillation ( $\omega$ ) for balloon.

[4]

Condition:

$\omega =$

- (f) Calculate approximately the time period ( $\tau$ ) of the oscillating balloon if  $T_0 = 300$  K in a balloon consisting of Ar-He mixture in the mass ratio 1:1 (Atomic Weights of Ar = 36 amu and He = 4 amu).

[2]

 $\tau =$ **\*\*\*\* END OF THE QUESTION PAPER \*\*\*\***

**Extra Sheet**

HB  
CSE



**Extra Sheet**

HB  
CSE

**Extra Sheet**

HB  
CSE

**Extra Sheet**

HB  
CSE

**Extra Sheet**

HB  
CSE