

GS-2014 (Physics)  
**TATA INSTITUTE OF FUNDAMENTAL RESEARCH**

Written Test in **PHYSICS - December 8, 2013**

Duration : Three hours (3 hours)

Name : \_\_\_\_\_ Ref. Code : \_\_\_\_\_

**Please read all instructions carefully before you attempt the questions.**

1. Please fill-in details about name, reference code etc. on the question paper and answer sheet. The Answer Sheet is machine-readable. Read the instructions given on the reverse of the answer sheet before you start filling it up. Use only HB pencils to fill-in the answer sheet.
2. Indicate your ANSWER ON THE ANSWER SHEET by blackening the appropriate circle for each question. Do not mark more than one circle for any question : this will be treated as a wrong answer.
3. This test consists of three parts, **Section A**, **Section B** and **Section C**. You must answer questions according to the instructions below :

Candidates applying for	Must Answer	Should Not Attempt
Integrated Ph.D.	Section A and Section B	Section C
Ph.D.	Section A and Section C	Section B

The test contains multiple choice-type questions. Only ONE of the options given at the end of each question is correct. Section A contains 25 questions, Section B and Section C contain 15 questions each.

Marking shall be as follows :

- (i) If the answer is **correct** : +3 marks
- (ii) If the answer is **incorrect** : -1 mark
- (III) If the answer is **not attempted** : 0 marks
- (iv) If **more than one box is marked** : 0 marks

*Note that negative marking as indicated above will be applicable.*

4. As a rough guideline, the time spent on questions in **Section A** should be about 5 minutes each; questions in **Section B and C** should take about 8 minutes each. Obviously, some questions may take a little less time while others may require a little more.
5. We advise you to first mark the correct answers on the QUESTION PAPER and then to TRANSFER these to the ANSWER SHEET only when you are sure of your choice.
6. Rough work may be done on blank pages of the question paper. If needed, you may ask for extra rough sheets from an Invigilator.
7. Use of calculators is permitted. Calculator which plots graphs is NOT allowed. Multiple-use devices such as cell phones, smartphones etc., CANNOT be used for this purpose.
8. Do NOT ask for clarifications from the invigilators regarding the questions. They have been instructed not to respond to any such inquiries from candidates. In case a correction/clarification is deemed necessary, the invigilator(s) will announce it publicly.
9. List of useful **physical constants** is given on the next page.

## USEFUL CONSTANTS

Symbol	Name/Definition	Value
$c$	speed of light in vacuum	$3 \times 10^8 \text{ m s}^{-1}$
$\hbar$	reduced Planck constant ( $= h/2\pi$ )	$1.04 \times 10^{-34} \text{ J s}$
$G_N$	gravitational constant	$6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
$\epsilon_0$	permittivity of free space	$8.85 \times 10^{-12} \text{ F m}^{-1}$
$\mu_0$	permeability of free space	$4\pi \times 10^{-7} \text{ N A}^{-2}$
$e$	electron charge (magnitude)	$1.6 \times 10^{-19} \text{ C}$
$m_e$	electron mass	$9.1 \times 10^{-31} \text{ kg}$ $= 0.5 \text{ MeV}/c^2$
$a_0$	Bohr radius	$0.51 \text{ \AA}$
	ionisation potential of H atom	$13.6 \text{ eV}$
$N_A$	Avogadro number	$6.023 \times 10^{23} \text{ mol}^{-1}$
$k_B$	Boltzmann constant	$1.38 \times 10^{-23} \text{ J K}^{-1}$ $= 8.62 \times 10^{-5} \text{ eV K}^{-1}$
$R = N_A k_B$	gas constant	$8.31 \text{ J mol}^{-1} \text{ K}^{-1}$
$\gamma = C_p/C_V$	ratio of specific heats: monatomic gas	1.67
	diatomic gas	1.40
$\sigma$	Stefan-Boltzmann constant	$5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
$\alpha$	fine structure constant ( $= e^2/4\pi\epsilon_0\hbar c$ )	1/137
$g$	acceleration due to gravity	$9.8 \text{ m s}^{-2}$
$R_E$	radius of the Earth	$6.4 \times 10^3 \text{ Km}$
$R_S$	radius of the Sun	$7 \times 10^5 \text{ Km}$
$m_p$	proton mass ( $\approx 2000 m_e$ )	$1.7 \times 10^{-27} \text{ kg}$ $= 938.2 \text{ MeV}/c^2$
$m_n$	neutron mass ( $\approx 2000 m_e$ )	$1.7 \times 10^{-27} \text{ kg}$ $= 939.6 \text{ MeV}/c^2$

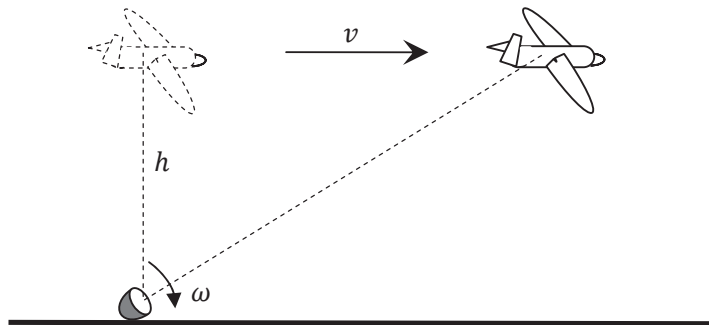
## UNIT CONVERSIONS

Symbol	Name/Definition	Value
1 A.U.	mean distance of Earth from Sun	$1.5 \times 10^9 \text{ km}$
1 a.m.u.	atomic mass unit	$1.6 \times 10^{-27} \text{ kg}$ $= 931.5 \text{ MeV}/c^2$
1 eV	electron Volt	$1.6 \times 10^{-19} \text{ J}$
$\hbar c$	conversion constant	$3.16 \times 10^{-26} \text{ J m}^{-1}$ $= 0.1973 \text{ GeV fm}$
1 T	Tesla	$10^4 \text{ gauss}$
1 bar	mean atmospheric pressure at $0^\circ \text{C}$	$1.01 \times 10^5 \text{ Pa} (= \text{N m}^{-2})$

## Section A

To be attempted by ALL candidates.

1. The directed beam from a small but powerful searchlight placed on the ground tracks a small plane flying horizontally at a fixed height  $h$  above the ground with a uniform velocity  $v$ , as shown in the figure below.



If the searchlight starts rotating with an instantaneous angular velocity  $\omega_0$  at time  $t = 0$  when the plane was directly overhead, then at a later time  $t$ , its instantaneous angular velocity  $\omega(t)$  is given by

(a)  $\frac{\omega_0}{1 + \omega_0^2 t^2}$  ✓

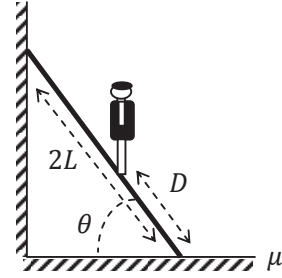
(b)  $\omega_0 \exp(-\omega_0 t)$

(c)  $\frac{\omega_0}{1 + \tan \omega_0 t}$

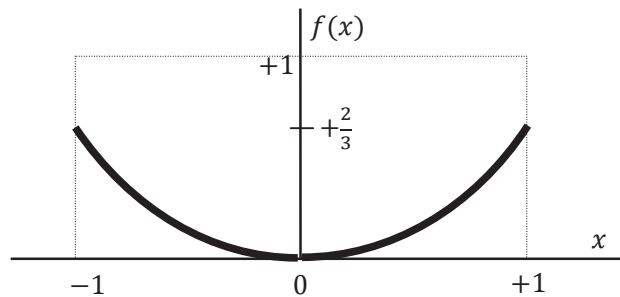
(d)  $\frac{\omega_0}{1 - \omega_0 t + \frac{1}{2}\omega_0^2 t^2}$

2. The Conservation Principles for energy, linear momentum and angular momentum arise from the necessity that
- the laws of physics should not involve infinite quantities.
  - internal forces on a body should cancel out, by Newton's (third) law of action and reaction.
  - the laws of physics should be independent of the state of rest or motion of the observer.
  - physical measurements should be independent of the origin and orientation of the coordinate system. ✓

3. A uniform ladder of length  $2L$  and mass  $m$  leans against a wall in a vertical plane at an angle  $\theta$  to the horizontal. The floor is rough, having a coefficient of static friction  $\mu$ . A person of mass  $M$  stands on the ladder at a distance  $D$  from its base (see figure). If the wall is frictionless, the maximum distance ( $D_{\max}$ ) up the ladder that the person can reach before the ladder slips is



- (a)  $\mu L \tan \theta$  (b)  $2\mu L \frac{m}{M} \tan \theta$   
(c)  $\left\{2\mu \left(1 + \frac{m}{M}\right) \tan \theta - \frac{m}{M}\right\} L$  ✓ (d)  $2\mu L \left(1 + \frac{m}{M}\right) \tan \theta$
4. The product  $\mathbf{MN}$  of two Hermitian matrices  $\mathbf{M}$  and  $\mathbf{N}$  is anti-Hermitian. It follows that  
(a)  $[\mathbf{M}, \mathbf{N}] = 0$  (b)  $\{\mathbf{M}, \mathbf{N}\} = 0$  ✓ (c)  $\mathbf{M}^\dagger = \mathbf{N}$  (d)  $\mathbf{M}^\dagger = \mathbf{N}^{-1}$
5. A student is asked to find a series approximation for the function  $f(x)$  in the domain  $-1 \leq x \leq +1$ , as indicated by the thick line in the figure below.



The student represents the function by a sum of three terms

$$f(x) \approx a_0 + a_1 \cos \frac{\pi x}{2} + a_2 \sin \frac{\pi x}{2}$$

Which of the following would be the best choices for the coefficients  $a_0$ ,  $a_1$  and  $a_2$  ?

- (a)  $a_0 = -\frac{1}{3}, a_1 = 0, a_2 = -1$  (b)  $a_0 = \frac{2}{3}, a_1 = 0, a_2 = -\frac{2}{3}$   
(c)  $a_0 = \frac{2}{3}, a_1 = -\frac{2}{3}, a_2 = 0$  ✓ (d)  $a_0 = 1, a_1 = -\frac{1}{3}, a_2 = 0$

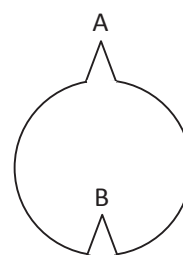
6. The probability function for a variable  $x$  which assumes only positive values is

$$f(x) = x \exp\left(-\frac{x}{\lambda}\right)$$

where  $\lambda > 0$ . The ratio  $\langle x \rangle / \hat{x}$ , where  $\hat{x}$  is the most probable value and  $\langle x \rangle$  is the mean value of the variable  $x$ , is

- (a) 1                      (b)  $\frac{1}{\lambda}$                       (c)  $\frac{1 + \lambda}{1 - \lambda}$                       (d) 2 ✓

7. A solid spherical conductor has a conical hole made at one end, ending in a point B, and a small conical projection of the same shape and size at the opposite side, ending in a point A. A cross-section through the centre of the conductor is shown in the figure on the right.



If, now, a positive charge  $Q$  is transferred to the sphere, then

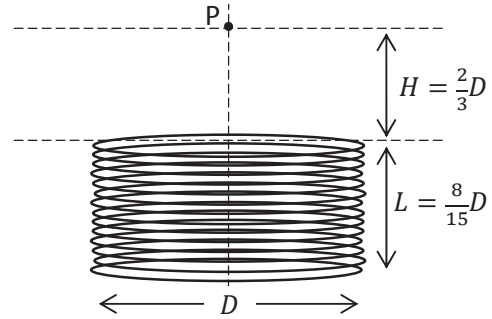
- (a) the charge density at A will be more than the charge density at B. ✓  
 (b) the charge density at B will be more than the charge density at A.  
 (c) the charge density at A will be the same as the charge density at B.  
 (d) the charge density at both A and B will be undefined.
8. Solving Poisson's equation  $\nabla^2 \varphi = -\rho_0 / \epsilon_0$  for the electrostatic potential  $\varphi(\vec{x})$  in a region with a constant charge density  $\rho_0$ , two students find different answers, viz.

$$\varphi_1(\vec{x}) = -\frac{\rho_0 x^2}{2\epsilon_0} \quad \text{and} \quad \varphi_2(\vec{x}) = -\frac{\rho_0 y^2}{2\epsilon_0}$$

The reason why these different solutions are both correct is because

- (a) space is isotropic and hence  $x$  and  $y$  are physically equivalent.  
 (b) the boundary conditions are different in the two cases. ✓  
 (c) we can add solutions of Laplace's equation to both  $\varphi_1(\vec{x})$  and  $\varphi_2(\vec{x})$ .  
 (d) the electrostatic energy is infinite for a constant charge density.

9. A short solenoid with  $n$  turns per unit length has diameter  $D$  and length  $L = 8D/15$ , as shown in the figure, and it carries a constant current  $I$ .



The magnetic field  $B$  at a point  $P$  on the axis of the solenoid at a distance  $H = 2D/3$  from its near end (see figure) is

[use  $\int dx (1 + x^2)^{-3/2} = x(1 + x^2)^{-1/2}$  ]

- (a)  $\frac{4}{13}\mu_0 nI$  (b)  $\frac{24}{15}\mu_0 nI$   
 (c)  $\frac{4}{65}\mu_0 nI$  ✓ (d)  $\frac{112}{65}\mu_0 nI$
10. A particle moving in one dimension has the un-normalised wave function

$$\psi(x) = x \exp\left(-\frac{x^2}{\lambda^2}\right)$$

where  $\lambda$  is a real constant. The expectation value of its momentum is  $\langle p \rangle =$

- (a)  $\frac{\hbar}{\lambda} \exp\left(-\frac{x^2}{\lambda^2}\right)$  (b)  $\frac{\hbar^2}{\lambda^2} - 2\frac{\hbar}{\lambda}$   
 (c)  $\frac{\hbar}{\lambda} \exp(-1)$  (d) zero ✓
11. A particle of mass  $m$  and charge  $e$  is in the ground state of a one-dimensional harmonic oscillator potential in the presence of a uniform external electric field  $E$ . The total potential felt by the particle is

$$V(x) = \frac{1}{2}kx^2 - eEx$$

If the electric field is suddenly switched off, then the particle will

- (a) make a transition to any harmonic oscillator state with  $x = -eE/k$  as origin without emitting any photon.  
 (b) make a transition to any harmonic oscillator state with  $x = 0$  as origin and absorb a photon. ✓  
 (c) settle into the harmonic oscillator ground state with  $x = 0$  as origin after absorbing a photon.  
 (d) oscillate back and forth with initial amplitude  $eE/k$ , emitting multiple photons as it does so.

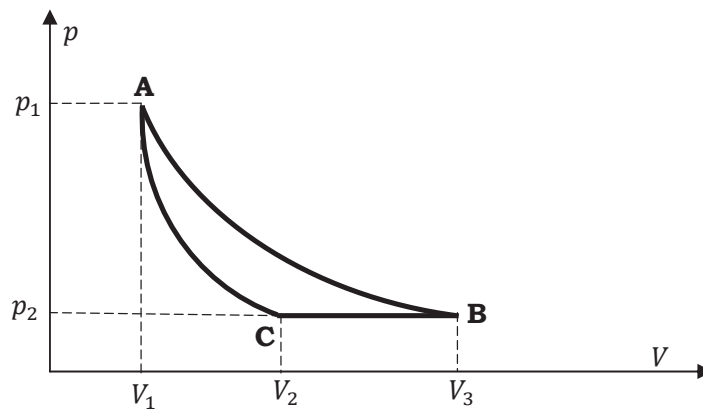
12. Consider the Hamiltonian

$$H = f \vec{\sigma} \cdot \vec{x}$$

Here  $\vec{x}$  is the position vector,  $f$  is a constant and  $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ , where  $\sigma_x, \sigma_y, \sigma_z$  are the three Pauli matrices. The energy eigenvalues are

- (a)  $f(\sqrt{x^2 + y^2} \pm z)$  (b)  $\pm f\sqrt{x^2 + y^2 + z^2}$  ✓  
 (c)  $\pm f(x + y + z)$  (d)  $f(x \pm iy)$

13. One mole of an ideal gas undergoes the cycle ACBA shown in the  $pV$  diagram below.



One of the curved lines in the cycle represents an isothermal change at temperature  $T$ , while the other represents an adiabatic change. The net heat gained by the gas in this cycle is

- (a)  $-p_2(V_3 - V_2) + RT \ln \frac{V_3}{V_1}$  ✓ (b)  $-p_2(V_3 - V_2) + RT \ln \frac{V_2}{V_1}$   
 (c)  $-p_2(V_3 - V_2) + \gamma RT(V_2^{1-\gamma} - V_1^{1-\gamma})$  (d)  $(p_1V_1 - p_2V_2) - RT \ln \frac{V_3}{V_1}$

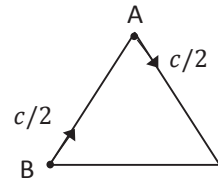
14. An ideal gas at a temperature  $T$  is enclosed in a rigid container whose walls are initially at temperature  $T_1$ , where  $T_1 < T$ . The walls are covered on the outside with perfect thermal insulation and the system is allowed to come to equilibrium. The pressure exerted by the gas on the walls of the container

- (a) remains constant throughout.  
 (b) is lower at the initial stage than at the final stage.  
 (c) is higher at the initial stage than at the final stage. ✓  
 (d) is the same at the initial and final stages.

15. Consider the CO molecule as a system of two point particles which has both translational and rotational degrees of freedom. Using classical statistical mechanics, the molar specific heat  $C_V$  of CO gas is given in terms of the Boltzmann constant  $k_B$  by

- (a)  $\frac{5}{2}k_B$  ✓ (b)  $2k_B$  (c)  $\frac{3}{2}k_B$  (d)  $\frac{1}{2}k_B$

16. In the laboratory frame, two observers A and B are moving along the sides of an equilateral triangle with equal speeds  $c/2$ , as shown in the figure.



The speed of B as measured by A will be

- (a)  $\frac{\sqrt{3}}{2}c$  (b)  $\frac{4}{3\sqrt{3}}c$  (c)  $\frac{\sqrt{5}}{3}c$  ✓ (d)  $\frac{\sqrt{13}}{7}c$

17. Two telescopes X and Y have identical objective lenses, but the single-lens eyepiece of X is converging whereas the single-lens eyepiece of Y is diverging. If the magnification  $M$  of these two telescopes for objects at infinity is the same, the lengths  $L_X$  and  $L_Y$  of the two telescopes (length of a telescope is defined as the distance between the objective lens and the eyepiece) must be in the ratio

$$L_X/L_Y =$$

- (a)  $\frac{2M+1}{2M-1}$  (b)  $\frac{2M-1}{M+1}$  (c)  $\frac{M-1}{M+1}$  (d)  $\frac{M+1}{M-1}$  ✓

18. A lens can be constructed using a flat circular glass plate whose refractive index  $n$  varies radially, i.e.  $n = n(r)$ , where  $r$  is the radial distance from the centre of the plate. In order to make a convex lens by this method  $n(r)$  should vary (in terms of positive constants  $n(0)$  and  $\alpha$ ) as

- (a)  $n(0) - \alpha/r^2$  (b)  $n(0) - \alpha r^2$  ✓ (c)  $n(0) - \alpha r$  (d)  $n(0) - \alpha/r$

19. A solid sample has the property that, when cooled below a certain temperature, it expels any small applied magnetic field from within the material. Which of the following best describes this sample in the cooled state?

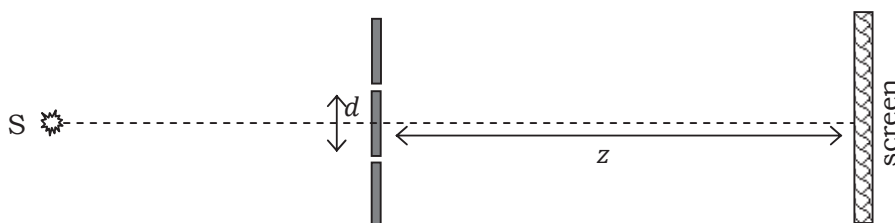
- (a) Ferromagnet (b) Anti-ferromagnet  
(c) Diamagnet ✓ (d) Paramagnet



20. A beam of atoms moving in a certain direction can be slowed down if they absorb photons from a laser beam moving in the opposite direction and subsequently spontaneously emit photons isotropically. For a beam of Sodium atoms (mass number  $A = 23$ ) with speed  $600 \text{ ms}^{-1}$ , if a laser beam of wavelength  $589 \text{ nm}$  is used, the number of such absorption and emission cycles needed to bring a Sodium atom to rest would be approximately

(a)  $2.1 \times 10^4$  ✓ (b)  $1.3 \times 10^4$  (c)  $2.1 \times 10^3$  (d)  $1.3 \times 10^5$

21. In a laboratory, the double-slit experiment is performed with free non-relativistic electrons, each having energy  $E$ , emitted from a source S (see figure below). The screen consists of a uniform sheet of charge-sensitive pixels of size  $r$ . If the slit-screen distance is  $z$  and the spacing between slits is  $d$ , which of the following restrictions on the electron energy  $E$  should be satisfied so that the fringes can be distinctly observed?



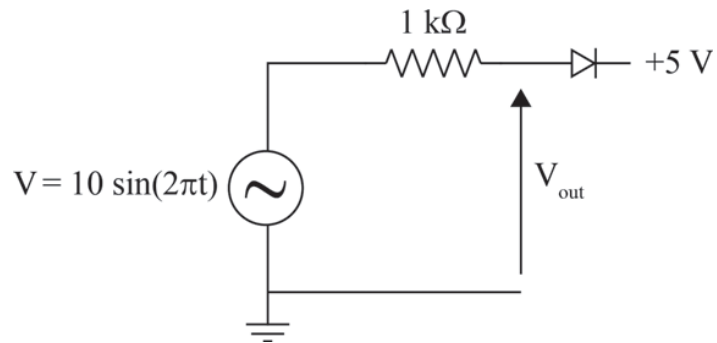
(a)  $E \geq \frac{1}{2m_e} \left( \frac{hz}{rd} \right)^2$  (b)  $E \leq \frac{1}{2m_e} \left( \frac{hz}{rd} \right)^2$  ✓

(c)  $E \leq c \left( \frac{hz}{rd} \right)$  (d)  $E \geq c \left( \frac{hz}{rd} \right)$

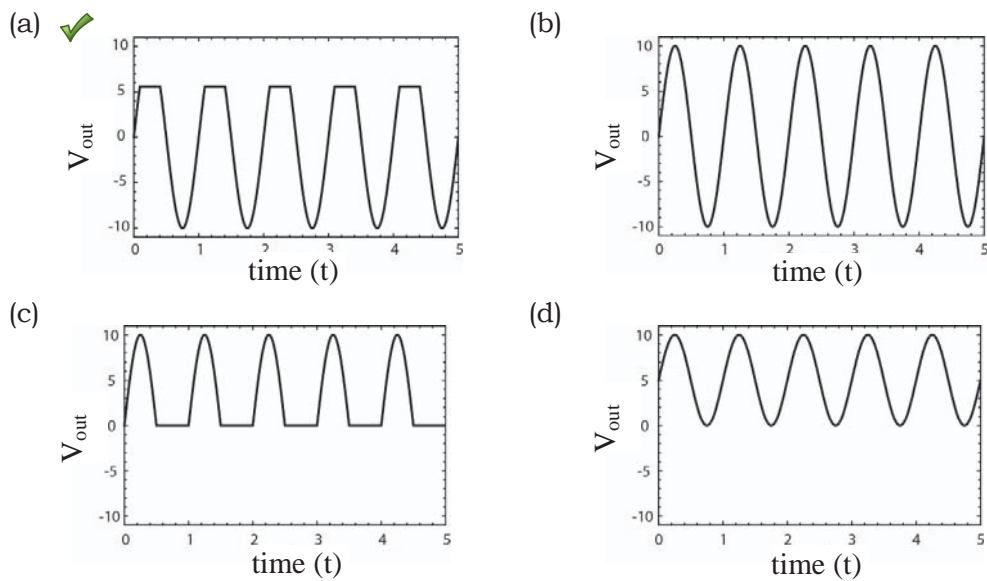
22. An alpha particle of energy  $E$  is shot towards a gold nucleus ( $^{197}_{79}\text{Au}$ ). At distances much larger than the nuclear size  $R_N$ , the dominant force is the Coulomb repulsion, but at distances comparable to the nuclear size the dominant force is the strong nuclear attraction. These combine to form a potential barrier of height  $V_C$ . If  $E < V_C$ , the probability that the alpha particle will fuse with the gold nucleus can be written (in terms of a dimensionless positive constant  $k$ ) as

(a) zero (b)  $k \left( 1 - \frac{E}{V_C} \right)$  (c)  $\exp \left( -\frac{kV_C}{E} \right)$  ✓ (d)  $\frac{kE}{\sqrt{k^2 E^2 + V_C^2}}$

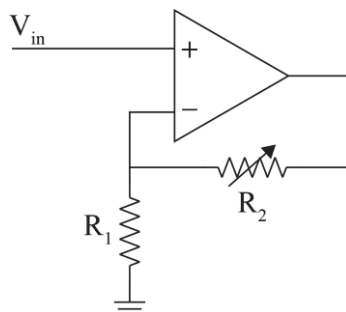
23. Consider the following circuit.



Which of the graphs given below is a correct representation of  $V_{out}$  ?



24. In the following circuit, the resistance  $R_2$  is doubled.



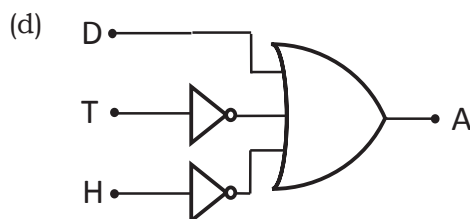
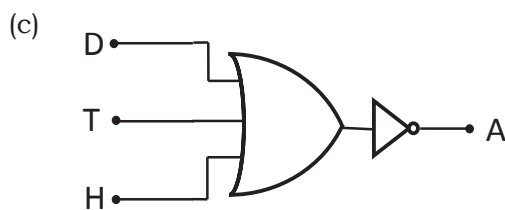
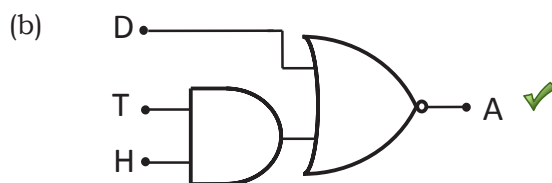
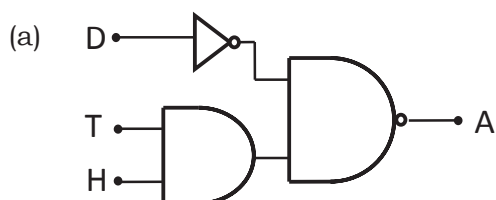
It follows that the current through  $R_2$

- (a) is halved.
- (b) is doubled.
- (c) is quadrupled.
- (d) remains the same. ✓

25. A control circuit needs to be designed to save on power consumption by an air-conditioning unit A in a windowless room with a single door. The room is fitted with the following devices:

1. a temperature sensor T, which is enabled ( $T = 1$ ) whenever the temperature falls below a pre-set value;
2. a humidity sensor H, which is enabled ( $H = 1$ ) whenever the humidity falls below a certain pre-set value;
3. a sensor D on the door, which is triggered ( $D = 1$ ) whenever the door opens.

Which of the following logical circuits will turn the air-conditioning unit off ( $A = 0$ ) whenever the door is opened or when *both* temperature and humidity are below their pre-set values?



## Section B

To be attempted only by candidates for Integrated Ph.D. programme.

(Candidates for Ph.D. programme will get NO credit for attempting this section.)

26. A body of mass  $m$  falls from rest at a height  $h$  under gravity (acceleration due to gravity  $g$ ) through a dense medium which provides a resistive force  $F = -kv^2$ , where  $k$  is a constant and  $v$  is the speed. It will hit the ground with a kinetic energy

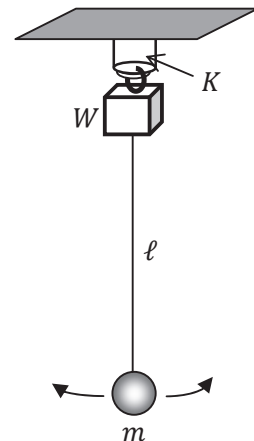
(a)  $\frac{m^2 g}{2k} \exp\left(-\frac{2kh}{m}\right)$

(b)  $\frac{m^2 g}{2k} \tanh \frac{2kh}{m}$

(c)  $\frac{m^2 g}{2k} \left\{1 - \exp\left(-\frac{2kh}{m}\right)\right\}$  ✓

(d)  $\frac{m^2 g}{2k} \left\{1 + \exp\left(-\frac{2kh}{m}\right)\right\}$

27. A weight  $W$  is suspended from a rigid support by a hard spring with stiffness constant  $K$ . The spring is enclosed in a hard plastic sleeve, which prevents horizontal motion, but allows vertical oscillations (see figure). A simple pendulum of length  $\ell$  with a bob of mass  $m$  ( $mg \ll W$ ) is suspended from the weight  $W$  and is set oscillating in a horizontal line with a small amplitude. After some time has passed, the weight  $W$  is observed to be oscillating up and down with a large amplitude (but not hitting the sleeve).



It follows that the stiffness constant  $K$  must be

(a)  $K = \frac{W}{2\ell}$

(b)  $K = \frac{2W}{\ell}$

(c)  $K = \frac{4W}{\ell}$  ✓

(d)  $K = \frac{W}{\ell}$

28. In spherical polar coordinates  $\vec{r} = (r, \theta, \varphi)$  the delta function  $\delta(\vec{r}_1 - \vec{r}_2)$  can be written as

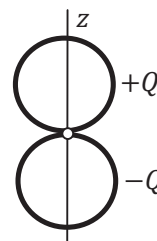
(a)  $\frac{1}{r_1^2} \delta(r_1 - r_2) \delta(\cos \theta_1 - \cos \theta_2) \delta(\varphi_1 - \varphi_2)$  ✓

(b)  $\frac{1}{r_1^2 \cos \theta_1} \delta(r_1 - r_2) \delta(\theta_1 - \theta_2) \delta(\varphi_1 - \varphi_2)$

(c)  $\frac{1}{|\vec{r}_1 - \vec{r}_2|^2} \delta(r_1 - r_2) \delta(\cos \theta_1 - \cos \theta_2) \delta(\varphi_1 - \varphi_2)$

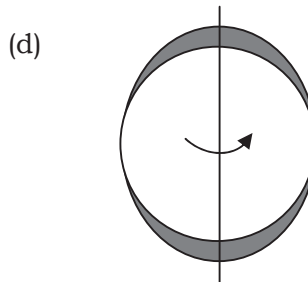
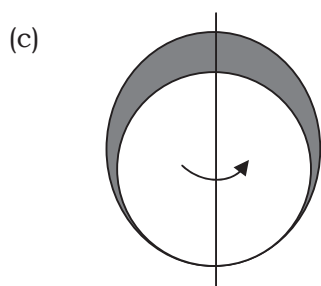
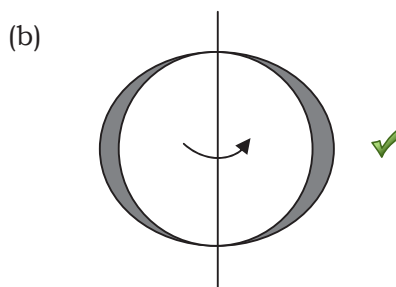
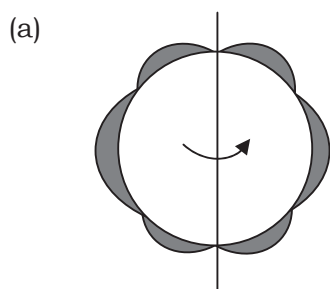
(d)  $\delta(r_1 - r_2) \delta(\theta_1 - \theta_2) \delta(\varphi_1 - \varphi_2)$

29. An electric dipole is constructed by fixing two circular charged rings, each of radius  $a$ , with an insulating contact (see figure). One of these rings has total charge  $+Q$  and the other has total charge  $-Q$ . If the charge is distributed uniformly along each ring, the dipole moment about the point of contact will be



- (a)  $2Qa \hat{z}$  ✓ (b)  $\frac{Qa}{\pi} \hat{z}$   
(c)  $4\pi Qa \hat{z}$  (d) zero

30. A spherical conductor, carrying a total charge  $Q$ , spins uniformly and very rapidly about an axis coinciding with one of its diameters. In the diagrams given below, the equilibrium charge density on its surface is represented by the thickness of the shaded region. Which of these diagrams is correct?



31. A rigid rotator is in a quantum state described by the wavefunction

$$\psi(\theta, \varphi) = \sqrt{\frac{3}{4\pi}} \sin \theta \sin \varphi$$

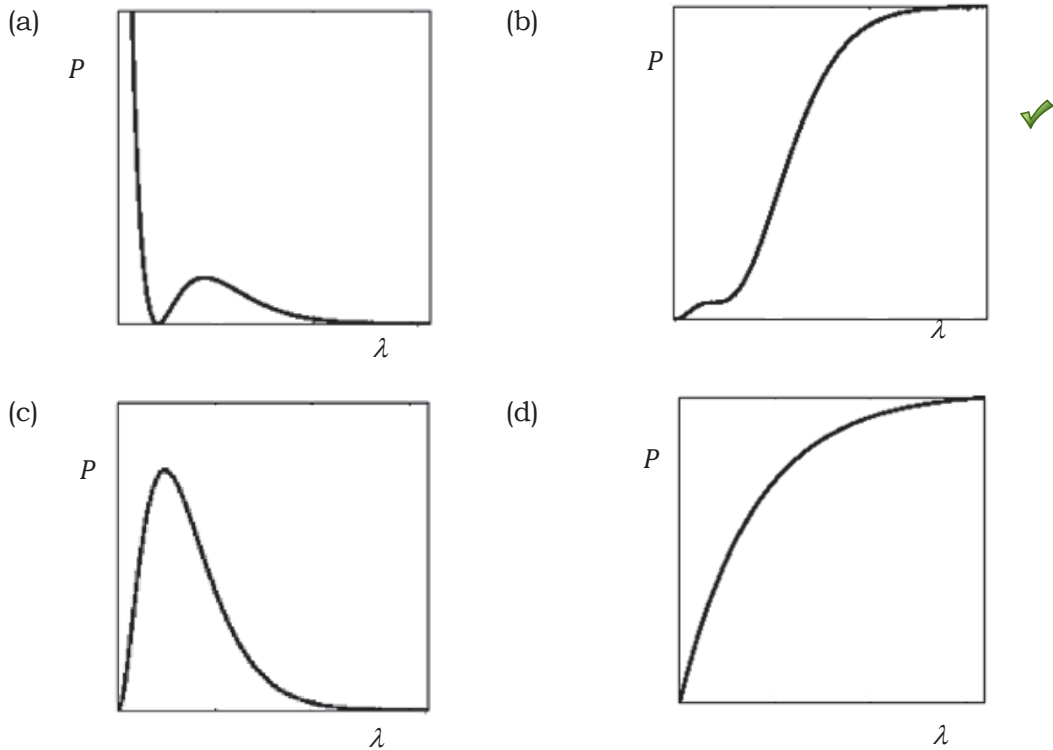
where  $\theta$  and  $\varphi$  are the usual polar angles. If two successive measurements of  $L_z$  are made on this rotator, the probability that the second measurement will yield the value  $+\hbar$  is

- (a) negligible (b) 0.25 (c) 0.33 (d) 0.5 ✓

32. A particle in the 2s state of hydrogen has the wave function

$$\psi_{2s}(r) = \frac{1}{4\sqrt{2}\pi} \left(\frac{1}{a_0}\right)^{3/2} \left(2 - \frac{r}{a_0}\right) \exp\left(-\frac{r}{2a_0}\right)$$

where  $r$  is the radial coordinate w.r.t. the nucleus as origin and  $a_0$  is the Bohr radius. The probability  $P$  of finding the electron somewhere inside a sphere of radius  $\lambda a_0$  centred at the nucleus, is best described by the graph



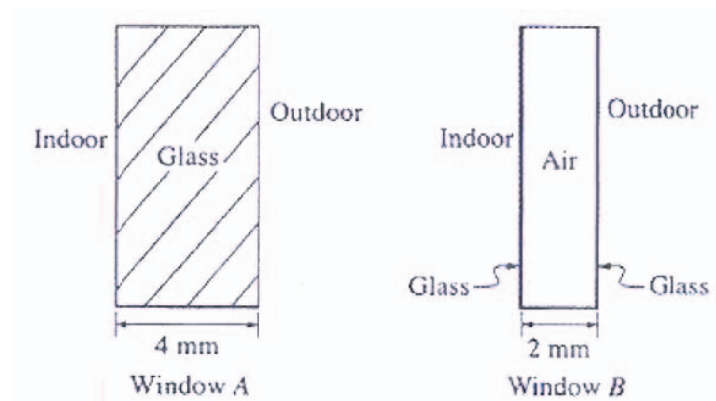
33. A thermally-insulated container of volume  $V_0$  is divided into two equal halves by a non-permeable partition. A real gas with equation of state

$$b^3 \left(p + \frac{a^2}{V^3}\right) = nRT$$

where  $a$  and  $b$  are constants, is confined to one of these halves at a temperature  $T_0$ . The partition is now removed suddenly and the gas is allowed to expand to fill the entire container. The final temperature of the gas, in terms of its specific heat  $C_V$ , will be

- (a)  $T_0 + \frac{3a^2}{2C_V V_0^2}$                       (b)  $T_0 - \frac{2a^2}{3C_V V_0^2}$
- (c)  $T_0 - \frac{3a^2}{2C_V V_0^2}$  ✓                      (d)  $T_0 + \frac{2a^2}{3C_V V_0^2}$

34. A manufacturer is able to offer two models of heat-conserving windows, as described below.



Window A is a simple pane of glass, 4 mm thick. Window B, on the other hand, consists of two extremely thin panes of glass, separated by an air gap of 2 mm, as shown in the figure above. If the thermal conductivity of glass is known to be  $0.8 \text{ Wm}^{-1}\text{K}^{-1}$  and that of air is  $0.025 \text{ Wm}^{-1}\text{K}^{-1}$ , then the ratio of heat flow  $Q_A$  through Window A to the heat flow  $Q_B$  through Window B is given by

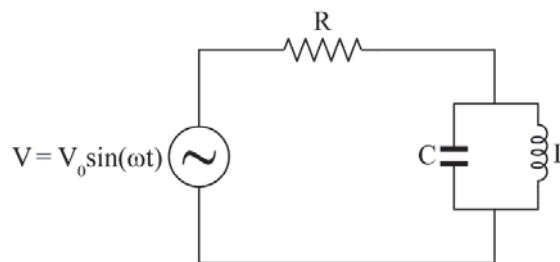
$$\frac{Q_A}{Q_B} =$$

- (a) 4                      (b) 16 ✓                      (c)  $\frac{1}{4}$                       (d)  $\frac{1}{16}$
35. In a low temperature experiment, the resistance of a sensor is used as a thermometer. In order to have better sensitivity in the range 100 mK to 1.0 K, which material would make the best sensor?
- (a) pure semiconductor ✓                      (b) p-n junction  
(c) insulator                      (d) metal
36. For a pure germanium semiconductor cooled in liquid nitrogen, the average density of conduction electrons is about  $n = 10^{12}$  per  $\text{cm}^3$ . At this temperature the electron and hole mobilities are equal and have the common value  $\mu = 5.0 \times 10^3 \text{ cm}^2 \text{ V}^{-1}\text{s}^{-1}$ . If a potential of 100 V is applied across a 1 cm cube of this cooled germanium sample, the current observed can be estimated as
- (a) 80  $\mu\text{A}$                       (b) 16 A  
(c) 16 mA                      (d) 160 mA ✓

37. A glass plate P (refractive index  $n_p = 1.54$ ) is coated with a dielectric material C with the refractive index  $n_c = 1.6$ . In order to have enhanced reflection from this coated glass for near-normal incident light of wavelength  $\lambda$ , the thickness of the coating material C must be

- (a) even multiples of  $\lambda/2n_c$       (b) even multiples of  $\lambda/4n_c$   
(c) odd multiples of  $\lambda/4n_c$  ✓      (d) integral multiples of  $\lambda/4n_c$

38. In the following circuit, the AC source is an ideal voltage source. What is the amplitude of the steady state current through the inductor at resonance?



- (a) zero      (b)  $V_0/R$   
(c)  $V_0 \sqrt{C/(R^2 C + 2L)}$       (d)  $V_0 \sqrt{C/L}$  ✓

39. A standard radioactive source is known to decay by emission of  $\gamma$  rays. The source is provided to a student in a thick sealed capsule of unbreakable plastic and she is asked to find out the half-life. Which of the following would be the most useful advice to the student?

- (a) Mount the source in front of a gamma ray detector and count the number of photons detected in one hour.  
(b) Mount the source in front of a gamma ray detector and count the number of photons detected in a specific time interval. Repeat this experiment at different times and note how the count changes. ✓  
(c) Measure the mass of the source at different times with an accurate balance having a least count of 1 mg. Plot these values on a curve and fit it with an exponential decay law.  
(d) The half-life cannot be measured because the initial concentration of the source is not given.



40. Cosmic ray muons generated at the top of the Earth's atmosphere decay according to the radioactive decay law

$$N(t) = N(0) \exp\left(-\frac{0.693 t}{T_{1/2}}\right)$$

where  $N(t)$  is the number of muons at time  $t$ , and  $T_{1/2} = 1.52 \mu\text{s}$  is the proper half-life of the muon. Immediately after generation, most of these muons shoot down towards the Earth's surface. Some of these muons decay on the way, but their interaction with the atmosphere is negligible.

An observer on the top of a mountain of height 2.0 km above mean sea level detects muons with the speed  $0.98c$  over a period of time and counts 1000 muons. The number of muons of the same speed detected by an observer at mean sea level in the same period of time would be

- (a) 539 ✓ (b) 983  
(c) 839 (d) 232

## Section C

To be attempted only by candidates for Ph.D. programme.

(Candidates for Integrated Ph.D. programme will get NO credit for attempting this section.)

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41. The integral

$$\int_0^{\infty} \frac{dx}{4+x^4}$$

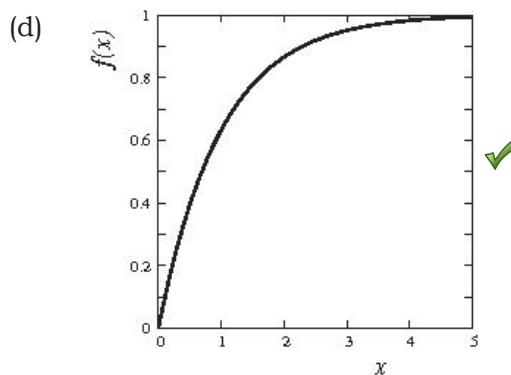
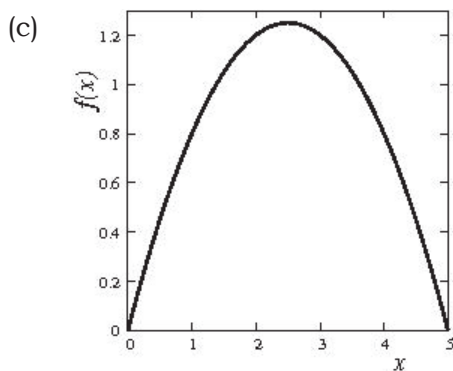
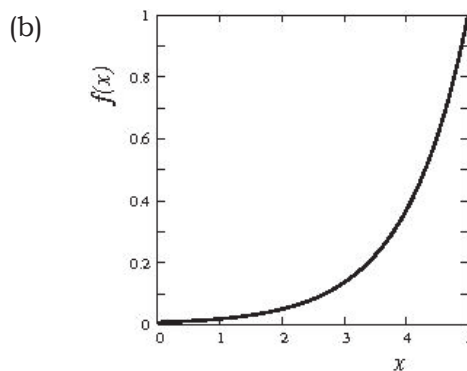
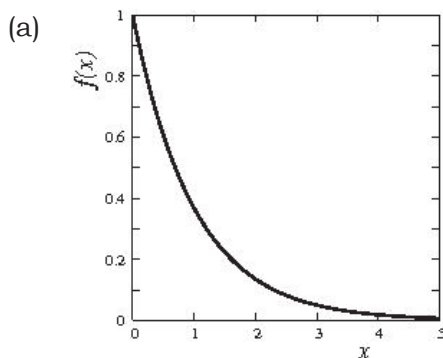
evaluates to

- (a)  $\pi$                       (b)  $\frac{\pi}{2}$                       (c)  $\frac{\pi}{8}$  ✓                      (d)  $\frac{\pi}{4}$

42. The solution of the integral equation

$$f(x) = x - \int_0^x dt f(t)$$

has the graphical form



43. Assume that the Earth is a uniform sphere of radius  $R$ , rotating about its axis with a uniform angular velocity  $\omega$ . A rocket is launched from the Equator in a direction due North. If it keeps on flying at a uniform speed  $v$  (neglecting air resistance), the highest latitude that can be achieved is

- (a)  $\frac{\pi}{2} - (\pi - 2) \frac{\omega R}{v}$  ✓  
 (b)  $\frac{\pi}{2} \left(1 - \frac{2\omega R}{v}\right)$   
 (c)  $\frac{\pi}{2} - (\pi + 2) \frac{\omega R}{v}$   
 (d)  $\frac{\pi}{2}$

44. A particle P of mass  $m$  moves under the influence of a central potential, centred at the origin O, of the form

$$V(r) = -\frac{k}{3r^3}$$

where  $k$  is a positive constant



If the particle P comes in from infinity with initial velocity  $u$  and impact parameter  $b$  (see figure), then the largest value of  $b$  for which the particle gets captured by the potential is

- (a)  $\left(\frac{2k^2}{m^2u^4}\right)^{1/6}$   
 (b)  $\left(\frac{k}{3mu}\right)^{1/3}$   
 (c)  $\left(\frac{3k^2}{m^2u^4}\right)^{1/6}$  ✓  
 (d)  $\left(\frac{2k}{3mu}\right)^{1/3}$

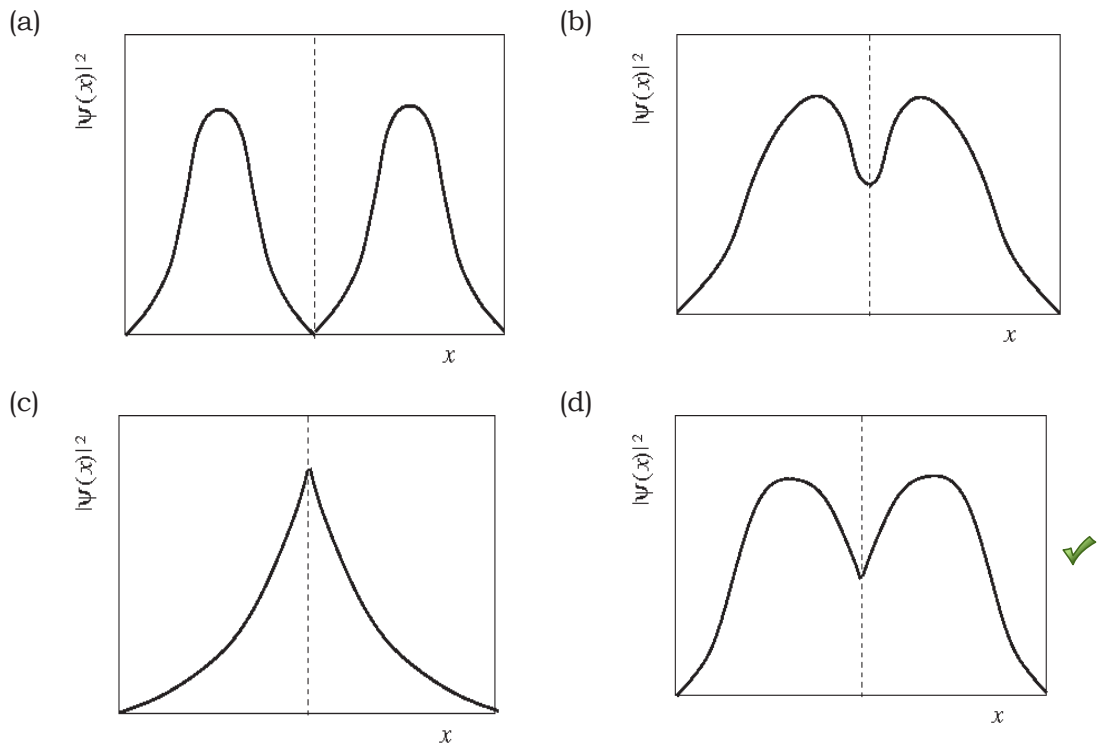
45. The instantaneous electric and magnetic fields created at a distance  $r$  by a point source at the origin are given by

$$\vec{E} = \frac{A \cos \omega t}{2\pi\epsilon_0 r} \hat{\theta} \quad \vec{H} = \frac{B \cos \omega t}{\mu_0 r} \hat{\phi}$$

where  $\omega, A, B$  are constants, and the unit vectors  $(\hat{r}, \hat{\theta}, \hat{\phi})$  form an orthonormal set. The time-averaged power radiated by the source is

- (a)  $\frac{\omega\epsilon_0}{\mu_0} AB$   
 (b)  $c^2 AB$  ✓  
 (c)  $\frac{c^3}{2\pi} AB$   
 (d)  $\frac{2\pi\omega}{c} AB$

46. A particle is confined to a one-dimensional box of length  $L$ . If a vanishingly thin but strongly repulsive partition is introduced in the exact centre of the box, and the particle is allowed to come to its ground state, then the probability density for finding the particle will appear as



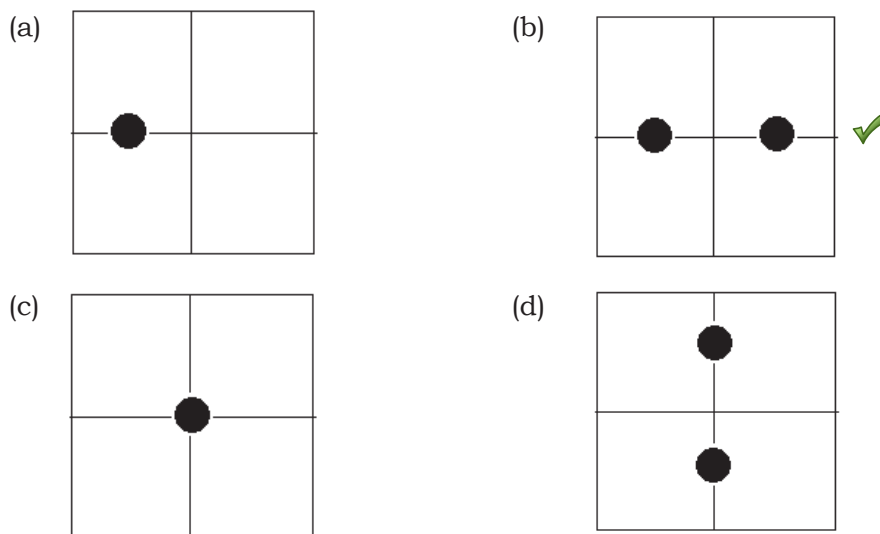
47. A spin-2 nucleus absorbs a spin- $\frac{1}{2}$  electron and is then observed to decay to a stable nucleus in two stages, recoiling against an emitted invisible particle in the first stage and against an emitted spin-1 photons in the second stage. If the stable nucleus is spinless, then the set of all possible spin values of the invisible particle is

- (a)  $\{1/2, 3/2, 5/2, 7/2\}$  ✓ (b)  $\{3/2, 7/2\}$   
(c)  $\{1/2, 3/2, 5/2\}$  (d)  $\{1/2, 5/2\}$

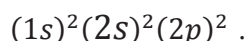
48. A gas of photons is enclosed in a container of fixed volume at an absolute temperature  $T$ . Noting that the photon is a massless particle (i.e., its energy and momentum are related by  $E = pc$ ), the number of photons in the container will vary as

- (a)  $T^3$  ✓ (b)  $T$  (c)  $T^2$  (d)  $T^4$

49. In a Stern-Gerlach experiment with spin- $\frac{1}{2}$  particles, the beam is found to form two spots on the screen, one directly above the other. The experimenter now makes a hole in the screen at the position of the upper spot. The particles that go through this hole are then passed through another Stern-Gerlach apparatus but with its magnets rotated by 90 degrees counter-clockwise about the axis of the beam direction. Which of the following shows what happens on the second screen?



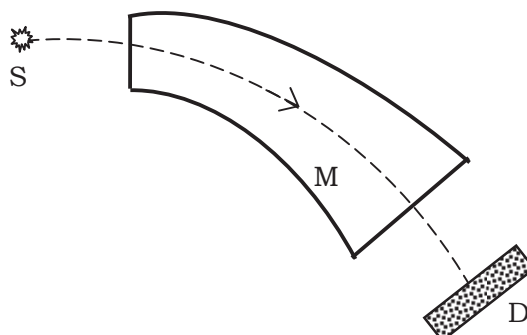
50. The ground state electronic configuration for a carbon atom is



The first excited state of this atom would be achieved by

- (a) transition of an electron from the  $2s$  orbital to the  $2p$  orbital.  
 (b) transition of an electron from the  $2p$  orbital to the  $3s$  orbital.  
 (c) transition of an electron from the  $2s$  orbital to the  $3s$  orbital.  
 (d) re-alignment of the electron spins within the  $2p$  orbital. ✓
51. Which of the following statement best explains why the specific heat of electrons in metals is much smaller than that expected in a non-interacting (free) electron gas model?
- (a) The mass of electron is much smaller than that of the ions in the crystal.  
 (b) Electrons in a metal cannot be modelled as non-interacting.  
 (c) Electron spin can take only two different values.  
 (d) The Pauli exclusion principle restricts the number of electrons which can absorb thermal energy. ✓

52. In a beta decay experiment, an electromagnet M and a detector D are used to measure the energy of electrons ( $\beta^-$ ), as shown in the figure.



The detector D is capable of detecting either electrons ( $\beta^-$ ) or positrons ( $\beta^+$ ). Now the  $\beta^-$  source is replaced with a  $\beta^+$  source, and we would like to measure the energy of the positrons ( $\beta^+$ ) using the same setup. Which of the following is correct?

- (a) There is no way to do this with the given set up, since  $\beta^+$  will have to be converted into  $\beta^-$ , which is obviously not possible.
- (b) This can be done trivially, without changing anything, since the detector D can detect either  $\beta^-$  or  $\beta^+$ .
- (c) This can be done quite easily, if the polarity of current in the coils of the electromagnet is reversed. ✓
- (d) This cannot be done since the magnet does not have a symmetric shape.
53. It is well-known that the energy of the Sun arises from the fusion of hydrogen nuclei (protons) inside the core of the Sun. This takes place through several mechanisms, each resulting in emission of energy.

Which of the following reactions is NOT possible during the proton fusion inside the Sun?

- (a)  ${}^1_1\text{H} + {}^1_1\text{H} \rightarrow {}^2_2\text{He}$
- (b)  ${}^1_1\text{H} + {}^1_1\text{H} + {}^1_1\text{H} + {}^1_1\text{H} \rightarrow {}^4_2\text{He} + 2e^+$  ✓
- (c)  ${}^2_1\text{H} + {}^1_1\text{H} \rightarrow {}^3_2\text{He}$
- (d)  ${}^1_1\text{H} + {}^1_1\text{H} \rightarrow {}^2_1\text{H} + e^+ + \nu_e$

54. A group of alien astronomers far away from the solar system tries to find out if the Sun (visible to them as a small yellow star) has planets orbiting around it. The method they use is to look for wobbles in the motion of the Sun induced by the planet(s) revolving around it (if any). To detect this motion, they build a high-resolution spectrometer which can measure the Doppler shift in frequency of a 600 nm line in the solar spectrum with an accuracy of 1 in  $10^6$ .

Given that the Sun has a mass  $2 \times 10^{30}$  kg and that Earth (mass  $6 \times 10^{24}$  kg, orbital velocity  $3 \times 10^4$  ms<sup>-1</sup>) and Jupiter (mass  $2 \times 10^{27}$  kg, orbital velocity  $1.5 \times 10^4$  ms<sup>-1</sup>) are two typical planets, one could predict that the experiments conducted by the aliens would find

- (a) no evidence for any planets orbiting the Sun. ✓
  - (b) evidence for the planet Jupiter, but not for the planet Earth.
  - (c) evidence for both the planets Earth and Jupiter.
  - (d) evidence for planets, but will not be able to tell how many.
55. The interaction strength of the recently-discovered Higgs boson (mass approximately  $125 \text{ GeV}/c^2$ ) with any other elementary particle is proportional to the mass of that particle. Which of the following decay processes will have the greatest probability?
- (a) Higgs boson decaying to a neutrino-antineutrino pair.
  - (b) Higgs boson decaying to an electron and a positron.
  - (c) Higgs boson decaying to a bottom quark + a bottom anti-quark. ✓
  - (d) Higgs boson decaying to a top quark + a top anti-quark.

**DO NOT WRITE ON THIS SHEET**