Part A
Common Questions

1. If either wages or prices are raised, there will be inflation. If there is inflation, then either the government must regulate it or the people will suffer. If the people suffer, the government will be unpopular. Government will not be unpopular. Which of the following can be validly concluded from the above statements.

   (a) People will not suffer

   (b) If the inflation is not regulated, then wages are not raised

   (c) Prices are not raised

   (d) If the inflation is not regulated, then the prices are not raised

   (e) Wages are not raised

2. In how many ways can the letters of the word ABACUS be rearranged such that the vowels always appear together?

   (a) $\frac{(6+3)!}{2!}$

   (b) $\frac{6!}{2!}$

   (c) $\frac{3!3!}{2!}$

   (d) $\frac{4!3!}{2!}$

   (e) None of the above

3. The probability of three consecutive heads in four tosses of a fair coin is

   (a) $\frac{1}{4}$

   (b) $\frac{1}{8}$

   (c) $\frac{1}{16}$

   (d) $\frac{3}{16}$

   (e) None of the above

4. Consider the problem of maximising $x^2 - 2x + 5$ such that $0 < x < 2$. The value of x at which the maximum is achieved is:

   (a) 0.5

   (b) 1

   (c) 1.5

   (d) 1.75

   (e) None of the above

5. Three distinct points x, y, z lie on a unit circle of the complex plane and satisfy $x + y + z = 0$. Then x, y, z form the vertices of

   (a) An isosceles but not equilateral triangle
   (b) An equilateral triangle
   (c) a triangle of any shape
   (d) a triangle whose shape can't be determined
   (e) None of the above

6. Assume that you are flipping a fair coin, i.e. probability of heads or tails is equal. Then the expected number of coin flips required to obtain two consecutive heads for the first time is

   (a) 4
   (b) 3
   (c) 6
   (d) 10
   (e) 5

7. Let X and Y be two independent and identically distributed random variables. Then $P(X > Y)$ is

   (a) $\frac{1}{2}$
   (b) 1
   (c) 0
   (d) $\frac{1}{3}$
   (e) Information is insufficient

8. The sum of the first n terms of the series $1, 11, 111, 1111, \cdots$, is

   (a) $\frac{1}{81}(10^{n+1} - 9n - 10)$
   (b) $\frac{1}{81}(10^{n} - 9n)$
   (c) $\frac{1}{9}(10^{n+1} - 1)$
   (d) $\frac{1}{9}(10^{n+1} - n10^{n})$
   (e) None of the above

9. You have to play three games with opponents A and B in a specified sequence. You win the series if you win two consecutive games. A is a stronger player than B. Which sequence maximizes your chance of winning the series?

   (a) AAB
   (b) ABA
   (c) BAB
   (d) BAA
   (e) All are the same

10. Let m, n denote two integers from the set $\{1, 2, \cdots, 10\}$. The number of ordered pairs (m, n) such that $2^m + 2^n$ is divisible by 5 is

   (a) 10
   (b) 14
   (c) 24
   (d) 8
   (e) None of the above

11. $\int_0^1 \log_e(x)dx =$

   (a) 1
   (b) -1
   (c) ∞
   (d) $-\infty$
   (e) None of the above

12. The action for this problem takes place in an island of knights and knaves, where knights always make true statements and knaves always make false statements and everybody is either a knight or a knave. Two friends A and B live in a house. The census taker (an outsider) knocks on the door and it is opened by A. The census taker says "I need information about you and your friend. Which if either is a knight and which if either is a knave?". "We are both knaves" says A angrily and slams the door. What, if anything can the census taker conclude?

   (a) A is a knight and B is a knave
   (b) A is a knave and B is a knight
   (c) Both are knaves
   (d) Both are knights
   (e) No conclusion can be drawn

13. If $z = \frac{\sqrt{3}-i}{2}$ and $(z^{95} + i^{67})^{97} = z^n$, then the smallest value of n is

(a) 1
(b) 10
(c) 11
(d) 12
(e) None of the above

14. The limit

$$\lim_{x \to 0} \frac{d}{dx} \frac{\sin^2(x)}{x}$$

is

(a) 0
(b) 2
(c) 1
(d) 1/2
(e) None of the above

15. The exponent of 3 in the product 100! is

(a) 27
(b) 33
(c) 44
(d) 48
(e) None of the above

16. A variable that takes thirteen possible values can be communicated using

(a) thirteen bits
(b) three bits
(c) $log_2 13$ bits
(d) four bits
(e) None of the above

17. What is

$$\lim_{x \to 0} \frac{2^x - 1}{x}$$

(a) 0

(b) $\log_2(e)$

(c) $\log_e(2)$

(d) 1

(e) None of the above

18. The equation of the tangent to the unit circle at point $(\cos \alpha, \sin \alpha)$ is

(a) $x \cos \alpha - y \sin \alpha = 1$

(b) $x \sin \alpha - y \cos \alpha = 1$

(c) $x \cos \alpha + y \sin \alpha = 1$

(d) $x \sin \alpha - y \cos \alpha = 1$

(e) None of the above.

19. Three dice are rolled independently. What is the probability that the highest and the lowest value differ by 4?

(a) 1/3

(b) 1/6

(c) 1/9

(d) 5/18

(e) 2/9

20. Let $n > 1$ be an odd integer. The number of zeros at the end of the number $99^n + 1$ is

(a) 1

(b) 2

(c) 3

(d) 4

(e) None of the above

Part B
Computer Science

21. Let $S = \{x_1, \ldots, x_n\}$ be a set of n numbers. Consider the problem of storing the elements of S in an array $A[1..n]$ such that the following min-heap property is maintained for all $2 \leq i \leq n$: $A[\lfloor i/2 \rfloor] \leq A[i]$. (Note that $\lfloor x \rfloor$ is the largest integer that is at most x.) Which of the following statements is TRUE?

    (a) This problem can be solved in $O(\log n)$ time.

    (b) This problem can be solved in $O(n)$ time but not in $O(\log n)$ time.

    (c) This problem can be solved in $O(n \log n)$ time but not in $O(n)$ time.

    (d) This problem can be solved in $O(n^2)$ time but not in $O(n \log n)$ time.

    (e) None of the above.

22. Consider the program P :: x:=1; y:=1; z:=1; u:=0
and the program Q :: x,y,z,u := 1,1,1,1; u:=0
Which of the following is true?

    (a) P and Q are equivalent for sequential processors.

    (b) P and Q are equivalent for all multi-processor models.

    (c) P and Q are equivalent for all multi-core machines.

    (d) P and Q are equivalent for all networks of computers.

    (e) None of the above.

23. Suppose $(S_1, S_2, \ldots, S_m)$ is a finite collection of non-empty subsets of a universe U. Note that the sets in this collection need not be distinct. Consider the following basic step to be performed on this sequence. While there exist sets S_i and S_j in the sequence, neither of which is a subset of the other, delete them from the sequence, and

    (i) if $S_i \cap S_j \neq \emptyset$, then add the sets $S_i \cup S_j$ and $S_i \cap S_j$ to the sequence;

    (ii) if $S_i \cap S_j = \emptyset$, then add only the set $S_i \cup S_j$ to the sequence.

    In each step we delete two sets from the sequence and add at most two sets to the sequence. Also, note that empty sets are never added to the sequence. Which of the following statements is TRUE?

    (a) The size of the smallest set in the sequence decreases in every step.

    (b) The size of the largest set in the sequence increases in every step.

    (c) The process always terminates.

    (d) The process terminates if U is finite but might not if U is infinite.

    (e) There is a finite collection of subsets of a finite universe U and a choice of S_i and S_j in each step such that the process does not terminate.

24. Consider the program
x:=0; y:=0; (r1:=x; r2:=x; y:= if r1 = r2 then 1 || r3:=y ; x:= r3)
Note that || denotes the parallel operator. In which of the following cases can the program possibly result in a final state with r1 = 0; r2 = r3 = 1.

(a) Such a transformation is not possible in Java.

(b) Such a program transformation is possible in Java.

(c) Possible in Pascal when the compiler appropriately translates the || operator to interleaved Pascal statements.

(d) Possible in all sequential programming languages when the compiler appropriately translates the || operator to interleaved statements in the sequential language.

(e) None of the above.

25. Let A_{TM} be defined as follows:

$$A_{TM} = \{\langle M, w\rangle | \text{The Turing machine } M \text{ accepts the word } w\}.$$

and let L be some NP-complete language. Which of the following statements is FALSE?

(a) $L \in NP$.

(b) Every problem in NP is polynomial time reducible to L.

(c) Every problem in NP is polynomial time reducible to A_{TM}.

(d) Since L is NP-complete, A_{TM} is polynomial time reducible to L.

(e) $A_{TM} \notin NP$.

26. Consider the following two scenarios in the dining philosophers problem: (i) First a philosopher has to enter a room with the table that restricts the number of philosophers to four. (ii) There is no restriction on the number of philosophers entering the room. Which of the following is true?

(a) Deadlock is possible in (i) and (ii).

(b) Deadlock is possible in (i).

(c) Starvation is possible in (i).

(d) Deadlock is not possible in (ii).

(e) Starvation is not possible in (ii).

27. Let n be a large integer. Which of the following statements is TRUE?

(a) $n^{\frac{1}{\sqrt{\log_2 n}}} < \sqrt{\log_2 n} < n^{\frac{1}{100}}$

(b) $n^{\frac{1}{100}} < n^{\frac{1}{\sqrt{\log_2 n}}} < \sqrt{\log_2 n}$

(c) $n^{\frac{1}{\sqrt{\log_2 n}}} < n^{\frac{1}{100}} < \sqrt{\log_2 n}$

(d) $\sqrt{\log_2 n} < n^{\frac{1}{\sqrt{\log_2 n}}} < n^{\frac{1}{100}}$

(e) $\sqrt{\log_2 n} < n^{\frac{1}{100}} < n^{\frac{1}{\sqrt{\log_2 n}}}$

28. Consider a basic block x:= a[i]; a[j]:=y; z:=a[j] optimized by removing common subexpression a[i] as follows: x:= a[i]; z:=x; a[j]:= y. Which of the following is true?

(a) Both are equivalent.

(b) The values computed by both are exactly the same.

(c) Both give exactly the same values only if i is not equal to j.

(d) They will be equivalent in concurrent programming languages with shared memory.

(e) None of the above.

29. You are given ten rings numbered from 1 to 10, and three pegs labeled A, B and C. Initially all the rings are on peg A, arranged from top to bottom in ascending order of their numbers. The goal is to move all the rings to peg B in the minimum number of moves obeying the following constraints:

i. In one move, only one ring can be moved.

ii. A ring can only be moved from the top of its peg to the top of a new peg.

iii. At no point can a ring be placed on top of another ring with a lower number.

How many moves are required?

(a) 501

(b) 1023

(c) 2011

(d) 10079

(e) None of the above

30. Consider an array A[1..n]. It consists of a permutation of numbers 1..n. Now compute another array B[1..n] as follows: B[A[i]]:= i for all i. Which of the following is true?

    (a) B will be a sorted array.

    (b) B is a permutation of array A.

    (c) Doing the same transformation twice will not give the same array.

    (d) B is not a permutation of array A.

    (e) None of the above

31. Given a set of $n = 2^k$ distinct numbers, we would like to determine the smallest and the second smallest using comparisons. Which of the following statements is TRUE?

    (a) Both these elements can be determined using $2k$ comparisons.

    (b) Both these elements can be determined using $n - 2$ comparisons.

    (c) Both these elements can be determined using $n + k - 2$ comparisons.

    (d) $2n - 3$ comparisons are necessary to determine these two elements.

    (e) nk comparisons are necessary to determine these two elements.

32. Various parameter passing mechanisms have been in used in different programming languages. Which of the following statements is true?

    (a) Call by value result is used in language Ada

    (b) Call by value result is the same as call by name.

    (c) Call by value is the most robust.

    (d) Call by reference is the same as call by name.

    (e) Call by name is the most efficient.

33. Which of the following is NOT a sufficient and necessary condition for an undirected graph G to be a tree?

    (a) G is connected and has $n - 1$ edges.

    (b) G is acyclic and connected.

    (c) G is acyclic and has $n - 1$ edges.

    (d) G is acyclic, connected and has $n - 1$ edges.

    (e) G has $n - 1$ edges.

34. Consider the class of synchronization primitives. Which of the following is false?

(a) Test and set primitives are as powerful as semaphores.

(b) There are various synchronizations that can be implemented using an array of semaphores but not by binary semaphores.

(c) Split binary semaphores and binary semaphores are equivalent.

(d) All statements a-c are false.

(e) Petri nets with and without inhibitor arcs have the same power.

35. Let G be a connected simple graph (no self-loops or parallel edges) on $n \geq 3$ vertices, with distinct edge weights. Let $e_1, e_2, \ldots, e_m$ be an ordering of the edges in *decreasing* order of weight. Which of the following statements is FALSE?

(a) The edge e_1 has to be present in every maximum weight spanning tree.

(b) Both e_1 and e_2 have to be present in every maximum weight spanning tree.

(c) The edge e_m has to be present in every minimum weight spanning tree.

(d) The edge e_m is never present in any maximum weight spanning tree.

(e) G has a unique maximum weight spanning tree.

36. Consider malware programs. Which of the following is true?

(a) A worm is a parasite

(b) A virus cannot affect a linux operating system.

(c) A trojan can be in the payload of only a worm.

(d) A worm and virus are self replicating programs

(e) There is no difference between a virus and a worm.

37. Given an integer $n \geq 3$, consider the problem of determining if there exist integers $a, b \geq 2$ such that $n = a^b$. Call this the *forward* problem. The *reverse* problem is: given a and b, compute $a^b \pmod{b}$. Note that the input length for the forward problem is $\lfloor \log n \rfloor + 1$, while the input length for the reverse problem is $\lfloor \log a \rfloor + \lfloor \log b \rfloor + 2$. Which of the following statements is TRUE?

(a) Both the forward and reverse problems can be solved in time polynomial in the lengths of their respective inputs.

(b) The forward problem can be solved in polynomial time, however the reverse problem is NP-hard.

(c) The reverse problem can be solved in polynomial time, however the forward problem is NP-hard.

(d) Both the forward and reverse problems are NP-hard.

(e) None of the above.

38. Consider the class of recursive and iterative programs. Which of the following is false?

   (a) Recursive programs are more powerful than iterative programs.
   (b) For every iterative program there is an equivalent recursive program.
   (c) Recursive programs require dynamic memory management.
   (d) Recursive programs do not terminate sometimes.
   (e) Iterative programs and recursive programs are equally expressive.

39. The first n cells of an array L contain positive integers sorted in decreasing order, and the remaining $m-n$ cells all contain 0. Then, given an integer x, in how many comparisons can one find the position of x in L?

   (a) At least n comparisons are necessary in the worst case.
   (b) At least $\log m$ comparisons are necessary in the worst case.
   (c) $O(\log(m-n))$ comparisons suffice.
   (d) $O(\log n)$ comparisons suffice.
   (e) $O(\log(m/n))$ comparisons suffice.

40. Consider the class of object oriented languages. Which of the following is true?

   (a) Pascal is an object oriented language.
   (b) Object oriented languages require heap management.
   (c) Object oriented languages cannot be implemented in language C.
   (d) Object oriented languages are more powerful than declarative programming languages.
   (e) Parallelism cannot be realized in object oriented languages.

Part C
Systems Science

41. Output of a linear system with input $x(t)$ is given by

$$y(t) = \int_{-\infty}^{\infty} h(t,\tau)x(\tau) + 1.$$

The system is linear if

(a) $h(t,\tau) = h(t-\tau)$
(b) $h(t,\tau) = h(\tau)$
(c) $h(t,\tau) = h(t)$
(d) $h(t,\tau) =$ constant
(e) None of the above.

42. The minimum number of unit delay elements required for realizing an infinite impulse response (IIR) filter is/are

(a) 0.
(b) 1.
(c) ∞.
(d) > 1.
(e) None of the above.

43. The Fourier transform of

$$x(t) = \frac{t^{n-1}}{(n-1)!} e^{-at} u(t), \quad a > 0$$

($\jmath = \sqrt{-1}$, $u(t) = 1$ for $t \geq 0$, $u(t) = 0$, $t < 0$) is

(a) $(a + \jmath\omega)^n$
(b) $\sum_{k=1}^{n} \frac{(a+\jmath\omega)^k}{k!}$
(c) $na\jmath\omega$
(d) $\frac{1}{(a+\jmath\omega)^n}$
(e) None of the above.

44. Let $\lim_{n\to\infty} x_n = x$. Then which of the following is TRUE.

(a) There exists an n_0, such that for all $n > n_0$, $|x_n - x| = 0$.
(b) There exists an n_0, such that for all $n > n_0$, $|x_n - x| \leq \epsilon$ for any $\epsilon > 0$.
(c) For every $\epsilon > 0$, there exists an n_0, such that for all $n > n_0$, $|x_n - x| \leq \epsilon$.
(d) There exists an n_0, such that for all $n > n_0$, $|\frac{x_n}{x}| \leq \epsilon$ for any $\epsilon > 0$.
(e) None of the above.

45. Consider a system with input $x(t)$ and the output $y(t)$ is given by

$$y(t) = x(t) - 0.5x(t-1) - 0.5x(t-2) + 1.$$

The system is

(a) Linear

(b) Non-causal

(c) Time varying

(d) All of the above

(e) None of the above

46. Let H(z) be the z-transform of the transfer function corresponding to an input output relation $y(n) - \frac{1}{2}y(n-1) = x(n) + \frac{1}{3}x(n-1)$. Then which of the following is TRUE

(a) The ROC of H(z) is $|z| > \frac{1}{2}$.

(b) The ROC of H(z) is $|z| < \frac{1}{2}$.

(c) Both (a) and (b).

(d) System is necessarily causal.

(e) None of the above.

47. Assume you are using a binary code error correcting code C. If the minimum Hamming distance between any two codewords of C is 3. Then

(a) We can correct and detect 2 bit errors.

(b) We can correct 1 bit errors and detect 2 bit errors.

(c) We can correct 2 bit errors and detect 1 bit errors.

(d) We can correct 1 bit errors and detect 1 bit errors.

(e) None of the above.

48. Let $f(x, y)$ be a function in two variables x, y. Then which of the following is true

(a) $\max_x \min_y f(x, y) \leq \min_y \max_x f(x, y)$.

(b) $\max_x \min_y f(x, y) \geq \min_y \max_x f(x, y)$.

(c) $\max_x \min_y f(x, y) = \min_y \max_x f(x, y)$.

(d) $\max_x \min_y f(x, y) = \min_y \max_x f(x, y) + \min_y \min_x f(x, y)$.

(e) None of the above.

49. Consider two independent random variables X and Y having probability density functions uniform in the interval $[-1, 1]$. The probability that $X^2 + Y^2 > 1$ is

(a) $\pi/4$
(b) $1 - \pi/4$
(c) $\pi/2 - 1$
(d) Probability that $X^2 + Y^2 < 0.5$
(e) None of the above

50. Let $f(x) = |x|$, for $x \in (-\infty,\ \infty)$. Then

(a) $f(x)$ is not continuous but differentiable.
(b) $f(x)$ is continuous and differentiable.
(c) $f(x)$ is continuous but not differentiable.
(d) $f(x)$ is neither continuous nor differentiable.
(e) None of the above.

51. What is the value of λ such that $\text{Prob}\{X > \text{mean}\{X\}\} = 1/e$, where PDF of X is $p_X(x) = \lambda e^{-\lambda x}$, $x \geq 0$, $\lambda > 0$?

(a) 1
(b) $1/e$
(c) $1/\sqrt{e}$
(d) $1/e^2$
(e) All of the above

52. Consider two communication systems C_1 and C_2 that use pulse amplitude modulation (PAM), PAM_1 and PAM_2. Let the distance between any two points of PAM_1 be d, and PAM_2 be $2d$, respectively. Assume that C_1 and C_2 are corrupted by additive white Gaussian noise of variance σ^2 and $2\sigma^2$, respectively. Let P_1 and P_2 be the probability of error for C_1 and C_2. Then

(a) $P_1 = P_2$.
(b) $P_1 < P_2$.
(c) $P_1 > P_2$.
(d) $P_1 = P_2 + \frac{1}{2}$.
(e) None of the above.

53. If a_k is an increasing function of k, i.e. $a_1 < a_2 < \ldots < a_k \ldots$. Then which of the following is TRUE.

(a) $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{a_k} = \infty$.

(b) $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{|a_k|} < \infty$.

(c) Either (a) or (b).

(d) $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{a_k} = 0$.

(e) None of the above.

54. In household electrical wiring which configuration is used to connect different electrical equipments.

(a) Series.

(b) Parallel

(c) Combination of series and parallel.

(d) Any of the above.

(e) None of the above.

55. Consider a channel where $x_n \in \{0, 1\}$ is the input and $y_n = x_n * z_n$ is the output, where $*$ is EX-OR operation, and $P(z_n = x_{n-1}) = P(z_n = y_{n-1}) = \frac{1}{2}$. Note that communication starts at time $n = 0$, and assume $x_{-1} = y_{-1} = 0$. Then the capacity of the channel in bits is

(a) $\frac{1}{2}$.

(b) 1.

(c) < 1.

(d) ≥ 0.

(e) Both (c) and (d).

56. Consider a triangular shaped pulse x of base $2T$ and unit height centered at 0, i.e. $x(t) = 0$ for $|t| > T$, $x(t) = 1 - |t|$ for $t \in [-T, T]$. Then if x is convolved with itself, the output is

(a) Square shape.

(b) Triangular shape.

(c) Bell shape.

(d) Inverted U shape.

(e) None of the above.

57. Let $x[n]$ and $y[n]$ be the input and output of a linear time invariant (LTI) system. Then which of following system is LTI.

   (a) $z[n] = y[n] + c$ for a constant c.

   (b) $z[n] = x[n]y[n]$.

   (c) $z[n] = y[n] + x[n] + c$ for a constant c.

   (d) $z[n] = y[n] + x[n]$.

   (e) None of the above.

58. Which of the following statements is TRUE.

   (a) The cascade of a non-causal linear time invariant (LTI) system with a causal LTI system can be causal.

   (b) If $h[n] \leq 2$ for all n, then the LTI system with $h[n]$ as its impulse response is stable and causal.

   (c) If the impulse response $h[n]$ of a LTI system is of finite duration then the LTI system is stable and causal.

   (d) With $h[n] = 3^n u[-n + 10]$, where $u[n] = 1, n \geq 0, u[n] = 0, n < 0$, the LTI system is stable.

   (e) Both (c) and (d).

59. Let $R_X(\tau)$ be the autocorrelation function of a zero mean stationary random process $X(t)$. Which of following statements is FALSE.

   (a) If $R_X(\tau) = 0,\ \forall\tau$, $X(n)$ and $X(m), n \neq m$ are independent.

   (b) $R_X(\tau) = R_X(-\tau)$.

   (c) $R_X(0) = E[X^2]$, where E denotes the expectation.

   (d) $R_X(0) \geq R_X(\tau),\ \forall\tau$.

   (e) None of the above.

60. Let $x(t)$ be a signal whose Fourier transform $X(f)$ is zero for $|f| > W$. Using a sampler with sampling frequency $4W$, which of the following filters can be used to exactly reconstruct $x(t)$.

   (a) Low pass filter spanning frequencies [-W W].

   (b) Band pass filter spanning frequencies [3W 5W].

   (c) Band pass filter spanning frequencies [-3W 5W].

   (d) All the above.

   (e) None of the above.