PAPER – III

MATHEMATICAL SCIENCES

Note : Attempt all the questions. Each question carries *two* (2) marks.

1. Let $S = \left\{\frac{1}{n} : n \in N\right\} \cup \{0\}$ and $T = \left\{n + \frac{1}{n}; n \in N\right\}$ be the subsets of the metric space R with the usual metric. Then

1) S is complete but not T
2) T is complete but not S
3) both T and S are complete
4) neither T nor S is complete

2. Let $f : [0, 1] \to R$ and $g : [0, 1] \to R$ be two functions defined by $f(x) = \begin{cases} \frac{1}{n}, \text{ if } x = \frac{1}{n}, n \in N \\ 0, \text{ otherwise} \end{cases}$ and $g(x) = \begin{cases} n, \text{ if } x = \frac{1}{n}, n \in N \\ 0, \text{ otherwise} \end{cases}$. Then

1) Both f and g are Riemann integrable
2) f is Riemann integrable but not g
3) g is Riemann integrable but not f
4) Neither f nor g is Riemann integrable

3. Let $\overline{E}$ is the set of point of closure of E

1) $\overline{E}$ is closed
2) $\overline{E}$ is open
3) $\overline{E}$ is null set
4) $\overline{E}$ is not closed

4. The power series $\sum_{n=0}^{\infty} \frac{[2 + (-1)^n]^n}{3^n} x^n$ converges

1) Only for $x = 0$
2) For all $x \in \mathbb{R}$
3) Only for $-1 < x < 1$
4) Only for $-1 < x \leq 1$

5. $\lim_{x \to 1} \frac{\sqrt{x^2 + 3} - 2\sqrt{x}}{x^2 - 1}$

1) $-\frac{1}{4}$
2) -1
3) 1
4) 0

6. Find reduced row-echelon form which is row-equivalent to the matrix $\begin{bmatrix} 0 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$

1) $\begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \end{pmatrix}$
2) $\begin{pmatrix} 0 & 1 & 3 \\ 1 & 0 & -2 \end{pmatrix}$
3) $\begin{pmatrix} 1 & 0 & 0 \\ 3 & 2 & -1 \end{pmatrix}$
4) $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \end{pmatrix}$

7. Dimension of the vector space of $m \times n$ matrices when trace is zero is

1) $n(n-1)$ 2) n^2

3) n^2+1 4) n^2-1

8. The number of linearly independent eigenvectors of matrix $\begin{bmatrix} 2 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is

1) 1 2) 2

3) 3 4) 4

9. The angle of rotation of $f(z)=z^2$ at $z_0=2+i$

1) $\tan^{-1}\left(\frac{1}{2}\right)$ 2) $\tan^{-1}\left(\frac{3}{4}\right)$

3) $\tan^{-1}(0)$ 4) $\tan^{-1}\left(\frac{1}{3}\right)$

10. Given $f(z)=\int_c \frac{z^2-z+1}{z-1}dz$, where C is a circle $|z|=1$

1) $f(z)$ is analytic in and on C 2) $f(z)=\frac{1}{2\pi i}$

3) $f(z)$ has simple poles 4) $f(z)$ is not analytic in and on C

11. Value of $\int \frac{ds}{y}$ is with respect to the transformation $z=\frac{az+b}{cz+d}$ where a, b, c, d satisfies $ad-bc=1$ and $ds=\sqrt{dx^2+dy^2}$

1) One 2) Variant

3) Zero 4) Invariant

12. The expansion of $f(z)=\frac{z}{(z-1)\,(z-3)}$ in $0<|z-1|<2$ is

1) $-\frac{1}{2(z-1)}-\frac{3}{4}\sum_{n=0}^{\infty}\left(\frac{z-1}{2}\right)^n$ 2) $\sum_{n=0}^{\infty}\frac{2^n-1}{2^n}Z^{n-1}$

3) $\frac{1}{2z}-\sum_{n=0}^{\infty}\frac{1}{z^{n+1}}-\frac{1}{4}\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n$ 4) $-\frac{1}{2(z-1)}-\frac{3}{4}\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n$

13. Let $X = \{1, 2, 3, 4, 5\}$ and $\tau = \{X, \Phi, \{2\}, \{3, 4\}, \{2, 3, 4\}, \{1, 3, 4\}, \{1, 2, 3, 4\}\}$ then the τ-exterior of $\{3\}$ is

1) $\{2\}$
2) $\{2, 3\}$
3) $\{2, 4\}$
4) Φ

14. A discrete space X is separable iff X is

1) Uncountable
2) Countable
3) Finite
4) Infinite

15. Let τ be the topology on N which consist of Φ and all subsets of N of the form $E_n = \{n, n+1, n+2, \ldots\}$ where $n \varepsilon N$, then the accumulation points of set A is

1) $A' = \{1, 2, 3, \ldots, 34, 35, 36\}$
2) $A' = \{1\}$
3) Φ
4) N

16. Consider the topology $\tau = \{X, \Phi, \{x\}, \{z, w\}, \{x, z, w\}, \{y, z, w, u\}\}$ on $X = \{x, y, z, w, u\}$ then the number of components of X is

1) 1
2) 2
3) 3
4) 4

17. Let $f : X \to Y$ and $g : X \to Y$ be continuous functions from a topological space X into a Hausdorff space Y. Then $A = \{x : f(x) = g(x)\}$ is

1) closed subset of X
2) open subset of X
3) not a closed subset of X
4) not a open subset of X

18. Every projection $\pi_i : X \to X_i$ on a product space $X = \prod_i X_i$ is

1) Only continuous
2) Bi-continuous
3) Only open
4) Neither continuous nor open

19. The projection map $\Pi_1 : X \times Y \to X$ is a closed map if

1) X is compact
2) Y is compact
3) X is connected
4) Y is connected

20. Bernoulli's differential equation is given by ————————.

1) $\dfrac{dy}{dx} + P(x)\, y = Q(x)\, y^n$
2) $\dfrac{dy}{dx} + y = Q(x)\, y^2 + R(x)$
3) $\dfrac{dy}{dx} + P(x)\, y = Q(x)\, y^2$
4) $\dfrac{dy}{dx} + P(x)\, y = Q(x)$

21. The Lagrange interpolating polynomial for

i	x_i	y_i
0	0	10
1	2	13
2	4	14

1) $10x^2 + 2x - \frac{1}{4}$

2) $-\frac{1}{4}x^2 + 2x + 10$

3) $10x^2 + \frac{1}{2}x + \frac{1}{4}$

4) $10x^2 - \frac{1}{4}x + \frac{1}{2}$

22. The plane curve of fixed perimeter and maximum area is

1) A circle

2) A rectangle

3) A square

4) An oval

23. For spherical pendulum, where the spherical bob is constrained to move on a sphere

1) Then the potential energy is $mgr\cos\theta$

2) Then the potential energy is $mgr\sin\theta$

3) Then the potential energy is $-mgr\cos\theta$

4) Then the potential energy is $-mgr\sin\theta$

24. The path on which a particle in the absence of friction, will slide from one point to another in the shortest time under the action of gravity is

1) The equation of the path is $\begin{array}{l} x = b(\phi - \sin\phi) \\ y = b(1 - \cos\phi) \end{array}$

2) A straight line

3) A rectangle

4) A square

25. Consider the integral $\int_0^1 y^2 dx \le c\int_0^1 y'^2\, dx$ for all 'y' there exist $y(0) = y(1) = 0$. Then the value of the smallest constant c is

1) π^2

2) 1

3) $\frac{1}{\pi^2}$

4) -1

26. The point of support of a simple pendulum moves on an ellipse fixed in the same plane with the parametric equation $x = a\cos\omega t$, $y = a\sin\omega t$, ω = constant. The Lagrange equation of motion of the bob for small oscillations is $\ddot{\theta} + \frac{g\theta}{l}F(t) = \frac{aw^2}{l}\cos\omega t$. Then the value of the function $F(t)$ is

1) $1 + \frac{bw^2}{g}\sin\omega t$

2) $1 - \frac{bw^2}{g}\cos\omega t$

3) $\frac{bw^2}{g}\cos\omega t$

4) $\frac{bw^2}{g}\sin\omega t$

27. In any system of particles, suppose we do not assume that the internal force come in pairs. Then the fact that the sum of internal force is zero follows from

1) Newton's Second law

2) Conservation of angular momentum

3) Conservation of energy

4) Principle of virtual energy

28. In spherical pendulum, a small bob of mass m is constrained to move on a smooth spherical surface, of radius R, R being the length of the pendulum, the equation of motion is

1) $mgR\cos^2\theta = 0$

2) $\ddot{\theta} - mR^2\cos^2\theta\dot{\phi}^2 = 0$

3) $mR^2\sin^2\theta\dot{\phi}^2 =$ constant

4) $mR^2\cos^2\theta\dot{\phi}^2 =$ constant

29. A continuous random variable X has the density function $f(x) = \frac{c}{1+x^2}$, $-\infty < x < \infty$ then the value of c is

1) $-\frac{1}{\pi}$

2) $\frac{2}{\pi}$

3) $\frac{1}{\pi}$

4) $\frac{-2}{\pi}$

30. Out of 800 families with 4 children each, how many families would be expected to have 2 boys and 2 girls. Assume equal probabilities for boys and girls

1) 750

2) 550

3) 300

4) 700

31. A problem in statistics is given to three students A, B and C whose chances of solving it are $\frac{1}{2}, \frac{3}{4}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved if all of them try independently?

1) $\frac{1}{2}$

2) $\frac{29}{32}$

3) $\frac{3}{4}$

4) $\frac{1}{4}$

32. Suppose a frequency distribution is skewed with a median of \$75 and a mode of \$80. Which of the following is a possible value for the mean of distribution?

1) \$86 2) \$91

3) \$64 4) \$75

33. A firm wishes to estimate with an error of not more than 0.03 and a level or confidence of 98%, the proportion of consumers that prefers its brand of household detergent. Sales reports indicate that about 0.20 of all consumers prefer the firm's brand. What is the requisite sample size?

1) 965 2) 975

3) 985 4) 995

34. The chi-square goodness of fit is based on

1) multinational distribution

2) hypergeometric distribution

3) the assumption that the character under study is normal

4) the assumption that the character under study is exponential

35. The maximum value of $z = 2x + 3y$ subject to the constraints $x + y \leq 30$, $y \geq 3$, $0 \leq y \leq 12$, $x - y \geq 0$ and $0 \leq x \leq 20$ is

1) 11 2) 72

3) 33 4) 36

36. Non degenerate basic solution of the system of equations $x + 2y + z = 4$, $2x + y + 5z = 5$ are

1) (2, 1, 0), (5, 0, –1) 2) (5, 0, –1), (0, 5/3, 2/3)

3) (2, 1, 0), (0, 5/3, 2/3) 4) (2, 1, 0), (0, 5/3, 3/2)

37. A branch of Punjab National Bank has only one typist. Since the typing work varies in length, the typing rate is randomly distributed approximating a Poisson distribution with mean service rate of 8 letters per hour. The letters arrive at a rate of 5 per hour during the entire 8-hour work day. If the computer is valued at Rs.1.50 per hour, find the average idle time cost of the computer per day

1) Rs. 4.20 2) Rs. 4.30

3) Rs. 4.40 4) Rs. 4.50

38. A fair die is tossed repeatedly. If X_n denotes the maximum of the numbers occurring in the first n tosses. Find the initial state probability distribution

1) $\left(\frac{1}{6}\ \frac{2}{6}\ \frac{3}{6}\ \frac{4}{6}\ \frac{5}{6}\ \frac{6}{6}\right)$ 2) $\left(\frac{1}{6}\ 0\ 0\ 0\ 0\ 0\right)$

3) $\left(\frac{1}{6}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{6}\right)$ 4) $\left(\frac{1}{6}\ 1\ 1\ 1\ 1\ 1\right)$

39. The sequence $\left\{(-1)^n + \frac{1}{n}\right\}_n$ is

1) Convergent
2) Uniformly convergent
3) Not Convergent
4) Conditionally convergent

40. Let $S = \left\{x \in \mathbb{R} : x \geq 0, \sum_{n=1}^{\infty} x^{\sqrt{n}} < \infty\right\}$. Then the supremum of S is

1) 1
2) $\frac{1}{e}$
3) 0
4) ∞

41. Let $f(x) = \frac{x^n}{1+x}$ and $g_n(x) = \frac{x^n}{1+nx}$ for $x \in [0, 1]$ and $x \in N$. Then on the interval $[0, 1]$

1) Both $\{f_n\}$ and $\{g_n\}$ converges uniformly
2) Neither $\{f_n\}$ nor $\{g_n\}$ converges uniformly
3) $\{f_n\}$ converges uniformly but $\{g_n\}$ does not converges uniformly
4) $\{g_n\}$ converges uniformly but $\{f_n\}$ does not converges uniformly

42. Which one of the following statements holds?

1) The series $\sum_{n=0}^{\infty} x^n$ converges for each $x \in [-1, 1]$
2) The series $\sum_{n=0}^{\infty} x^n$ converges uniformly in $x \in [-1, 1]$
3) The series $\sum_{n=0}^{\infty} \frac{x^n}{n}$ converges for each $x \in [-1, 1]$
4) The series $\sum_{n=0}^{\infty} \frac{x^n}{n}$ converges uniformly in $x \in [-1, 1]$

43. Consider the function $f : R \to R$ defined by $f(x) = \begin{cases} x^4, & \text{if } x \text{ is rational} \\ 2x^2 - 1, & \text{if } x \text{ is irrational} \end{cases}$.

Let S be the set of points where f is continuous. Then

1) $S = \{1\}$
2) $S = \{-1\}$
3) $S = \{-1, 1\}$
4) $S = \phi$

44. If M is a 7×5 matrix of rank 3 and N is a 5×7 matrix of rank 5, then rank (MN) is

1) 5
2) 3
3) 2
4) 1

45. Let M be a skew symmetric, orthogonal real matrix. The only possible eigenvalues

1) $(-1, 1)$
2) $(-i, i)$
3) 0
4) $(1, i)$

46. Consider the matrix $A=\begin{bmatrix}1 & 2 & 5\\ 0 & 1 & 7\\ 0 & 0 & 0\end{bmatrix}$. Then

1) $\dim\ col(A)=1$
2) $\dim$ Null $(A)=2$
3) rank $(A)=3$
4) rank $(A)=2$

47. Let $T\in L(R^2)$ be defined by $T\begin{pmatrix}a\\ -b\end{pmatrix}=\begin{pmatrix}b\\ -a\end{pmatrix}$ for all $\begin{pmatrix}a\\ b\end{pmatrix}\in R^2$. Then which one of the following is false?

1) T is surjective
2) $\dim(null\ (T))=0$
3) The map $F: R^2\to R^2$ given by $F(x,y)=(x+y,\ x+1)$ is not linear
4) T is not surjective

48. Which of the following is diagonalizable?

1) $\begin{bmatrix}2 & 1 & -1\\ 1 & -3 & 2\\ -1 & 2 & 0\end{bmatrix}$
2) $\begin{bmatrix}2 & 1 & 0\\ 0 & 2 & 0\\ 0 & 0 & -1\end{bmatrix}$
3) $\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 5\\ 0 & 0 & 6\end{bmatrix}$
4) $\begin{bmatrix}4 & 1 & 0\\ 0 & 4 & 0\\ 0 & 0 & 2\end{bmatrix}$

49. If $T: R^2\to R^3 \ni T(x+y)=(x+y,\ 0,\ x-y)$

1) T is linear
2) $N(T)\neq 0$
3) T is a bijection
4) $N(T)=3$

50. Let $\gamma(t) = e^{it}, \ 0 \le t \le 2\pi$

1) $\int_\gamma \sin\left(\frac{1}{z}\right) dz = 0$

2) $\int_\gamma \sin^2\left(\frac{1}{z}\right) dz = 1$

3) $\int_\gamma \sin\frac{1}{z} dz = 2\pi i$

4) $\int_\gamma \sin^2\left(\frac{1}{z}\right) dz = 2\pi i$

51. If *f* is an entire function such that $f(z) \to \infty$ as $z \to \infty$ then

1) *f* has a residue at $z = 0$

2) *f* has a simple pole at $z = \infty$

3) *f* has an essential singularity at $z = \infty$

4) *f* has a pole at $z = \infty$ but need not be simple

52. Let G be a group of order 231. The number of elements of order 11 in G is ————.

1) 10
2) 12
3) 11
4) 2

53. A sylow 3-subgroup of a group of order 12 has order

1) 3
2) 4
3) 12
4) 8

54. The number of homomorphism from z_{10^5} to z_{100} are

1) 100
2) 200
3) 30
4) 40

55. How many positive integers not exceeding 1000 are divisible by 7 or 11?

1) 142
2) 90
3) 220
4) 12

56. The remainder when 9^{10} is divided by 11 is

1) 1
2) 2
3) 5
4) 0

57. The primitive root $\bmod 18$ is ————.

1) 3
2) 5
3) 6
4) 9

58. The initial value problem $\frac{dy}{dt} = y^{\frac{1}{3}}$, $y(t_0) = 0$ has

1) a unique solution
2) two solutions
3) solution does not exist
4) at least three solutions

59. The general solution of the equation $\sin x\, dx + \frac{dy}{\sqrt{y}} = 0$ is

1) $2\sqrt{y} + \cos x = c$
2) $2\sqrt{y} - \cos x = c$
3) $2\sqrt{y} = \sin x$
4) $\sin x + \cos x = \sqrt{y}$

60. The complete integral of the PDE $z^2 = pqxy$ is

1) $z = bx^{1/a} y^a$
2) $z = bx^{1/a} y^{1/a}$
3) $z = bx^a\, y^{1/a}$
4) $z = bx^a\, y^a$

61. The canonical form of the PDE $u_{xx} + x^2\, u_{yy} = 0$ is

1) $u_{\alpha\alpha} + \alpha^2\, u_{\beta\beta} = 0$
2) $u_{\alpha\alpha} + u_{\beta\beta} = 0$
3) $u_{\alpha\alpha} + \alpha^2\, u_{\beta\beta} = -u_\alpha\, /(2\alpha)$
4) $u_{\alpha\alpha} + \alpha^2{}_{\beta\beta} = u_\alpha\, /(2\alpha)$

62. Using the transformation $u = W / y$ in the PDE $xu_x = u + yu_y$, the transformed equation has a solution of the form $W =$

1) $f(x / y)$
2) $f(x + y)$
3) $f(x - y)$
4) $f(xy)$

63. Find the particular integral for $(\Delta^2 + 2\Delta + 1)\, u_n = 3x$

1) $3x + 8$
2) $3x - 6$
3) $3x - 3$
4) $3x$

64. The Lagrange interpolating polynomial for the data

i	x_i	y_i
0	0	1
1	1	3
2	2	7

1) $x^2 - x + 1$
2) $x^2 + x + 1$
3) $x + 1$
4) $x^2 - 1$

65. The solution of the integral equation $S = \int_0^s \frac{g(t)}{(s-t)^{1/2}} dt$ is

1) $g(t) = \frac{2t^{1/2}}{\pi}$

2) $g(t) = \frac{3t^{1/2}}{\pi}$

3) $g(t) = \frac{t^2}{\pi}$

4) $g(t) = -\frac{t^2}{\pi}$

66. The resolvent kernel of integral equation $\phi(x) = \sin x + 2\int_0^x e^{x-t}\phi(t)dt$ is

1) $e^{2(x-t)}$

2) $e^{3(x-t)}$

3) $e^{(x-t)}$

4) $e^{(x+t)}$

67. Given the homogeneous integral equation $\phi(x) = \lambda \int_1^2 \left(x\xi + \frac{1}{x\xi} \right) \phi(\xi)\, d\xi$

1) Not a Fredholm integral equation

2) The eigen function of the system is $\lambda^2 + 13\lambda + 6 = 0$

3) The required eigen values are $\lambda_1 = \frac{1}{2}[17 + \sqrt{265}] = 6.639$ and $\lambda_2 = \frac{1}{2}[17 - \sqrt{265}] = 0.3606$

4) The eigen values are 0.037 and 3.3259

68. The principal moment of inertia for a homogeneous sphere of radius R and mass M is

1) $\frac{3}{5}MR^2$

2) $\frac{1}{2}MR^2$

3) $\frac{2}{5}MR^2$

4) $\frac{3}{2}MR^2$

69. A frame of reference in which law of inertia holds

1) Non-Newtonian frame of reference

2) Accelerated frame of reference

3) Newtonian frame of reference

4) Galilean frame of reference

70. The probability that John hits a target is $\frac{1}{2}$. He fixes 6 times. Find the probability that he hits the target exactly 2 times.

1) $\frac{15}{2^6}$ 2) $\frac{7}{2^6}$

3) $\frac{5}{2^6}$ 4) $\frac{13}{2^6}$

71. If X has the probability density $f(x) = ke^{-3x}$, $x > 0$ find k

1) –4 2) 4

3) –3 4) 3

72. If X follows $B\left(3, \frac{1}{3}\right)$ and Y follows $B\left(5, \frac{1}{3}\right)$ find $P(X + Y \geq 1)$

1) $1 + \left(\frac{2}{3}\right)^8$ 2) $1 - \left(\frac{2}{3}\right)^8$

3) $1 + \left(\frac{3}{4}\right)^8$ 4) $1 - \left(\frac{3}{4}\right)^8$

73. 3 mangoes and 3 apples are in a box. If fruits are chosen at random, the probability that one is a mango and the other is an apple, is

1) $\frac{3}{5}$ 2) $\frac{5}{6}$

3) $\frac{1}{36}$ 4) $\frac{3}{7}$

74. If A and B are tow events, the probability that exactly one of them occurs is given by

1) $P(A) + P(B) - 2P(A \cap B)$ 2) $P(A \cap \overline{B}) + P(\overline{A} \cap B)$

3) $P(A \cup B) - P(A \cap B)$ 4) $P(\overline{A}) + P(\overline{B}) - 2P(\overline{A} \cap \overline{B})$

75. A die was rolled 30 times with the results shown below :

Number of spots	1	2	3	4	5	6
Frequency	1	4	9	9	2	5

If a chi-square goodness of fit test is used to test the hypothesis that the die is fair at a significance level of $\alpha = 0.05$, then the value of the chi-square statistic and the decision reached are

1) 11.6 ; reject hypothesis 2) 11.6 ; accept hypothesis

3) 22.1 ; reject hypothesis 4) 22.1 : accept hypothesis

ROUGH WORK

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