PAPER – II

MATHEMATICAL SCIENCES

Note : Attempt all the questions. Each question carries *two* (2) marks.

1. Subset of R which is a neighborhood of 3 is

1) [3, 6]
2) [3, 6)
3) (2, 4)
4) (3, 6)

2. The series $\sum_{r=1}^{\alpha}(-1)^{r-1}$

1) Oscillates finitely
2) Divergent
3) Convergent
4) Oscillates infinitely

3. The sequence $\left[\dfrac{\cos\frac{n\pi}{2}}{n}\right]_{n=1}^{\infty}$ is ______________.

1) convergent to 0
2) divergent
3) convergent to 1
4) convergent to $\dfrac{1}{2}$

4. The radius of convergence of the series $\dfrac{x+.2}{1}+\dfrac{(x+.2)^2}{2}+\cdots+\dfrac{(x+.2)^n}{n}+\cdots$

1) 1
2) ∞
3) 0
4) $\dfrac{1}{2}$

5. Composite number n is ______________.

1) a prime number and $n > 1$

2) non-prime number and $n < 1$

3) non-prime number and $n > 1$

4) a prime number and $n < 1$

6. A function $f(x)$ has no jump discontinuity at $x = a$ if ______________.

1) $f(a+) = f(a-) = f(a)$

2) $f(a+) \neq f(a-)$

3) $f(a+) \neq f(a)$

4) $f(a+) = f(a-) \neq f(a)$

7. A subset S of a vector space V satisfying $V = L(S)$ is a basis if ______________.

1) S is linearly dependent

2) S is linearly independent

3) V is a field

4) S is a field

8. The value of the determinant $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$ is

1) abc

2) $a^2b^2c^2$

3) $bc + ca + ab$

4) zero

9. If $\dim W = m$, $\dim V = n$ and $W \subset V$ then $\dim(V/W)$ is ______________.

1) $m + n$

2) $n - m$

3) $m - n$

4) mn

10. The product of two orthogonal matrices is orthogonal and that the inverse of an orthogonal matrix is

1) Symmetric
2) Orthogonal
3) Skew-symmetric
4) Hermitian

11. Let V be an n-dimensional vector space and let $T : V \to V$ be a linear map such that $\text{null}(T) = \text{range}(T)$. Then

1) n is odd
2) n is even
3) n is neither odd or even
4) n is not defined

12. Every square matrix satisfies its own characteristic equation. This is

1) Cauchy's theorem
2) Cayley-Hamilton theorem
3) Eigen value theorem
4) Sylow's theorem

13. A function $f(z) = \text{Re}(z)$ is

1) analytic
2) nowhere differentiable
3) continuous
4) discontinuous

14. The real part of $\exp(\exp i\theta)$ is

1) $e^{\cos\theta}$
2) $e^{\cos\theta}\sin(\sin\theta)$
3) $e^{\cos\theta}\cos(\cos\theta)$
4) $e^{\cos\theta}\cos(\sin\theta)$

15. The value of $\lim_{z \to 0} \frac{\bar{z}}{z}$ is

1) 0
2) 1
3) 1/2
4) Limit does not exist

16. The fixed points of the bilinear transformation $w = \frac{z}{2-z}$ are

1) 0, 0
2) 0, 1
3) 0, 1/2
4) 1, 1/2

17. The primitive roots *modulo* 19 is ——————.

1) 18
2) 6
3) 5
4) 12

18. In the ring of even integers $2Z$, the ideal $I = <4>$ is

1) Integral domain
2) Principal ideal
3) Maximal but not prime
4) Maximal and prime

19. If D is an integral domain and $D[x]$ is a principal ideal domain, then D becomes a

1) Ring
2) Field
3) Integral domain
4) Ideal

20. If $\pi(N)$ denotes the number of prime numbers less than or equal to N then $\pi(6) =$

1) 2
2) 5
3) 1
4) 4

21. Which of the following statement is wrong?

1) Every subspace of discrete space is also discrete
2) Every subspace of an indiscrete space is indiscrete
3) Every non-empty open subset of an indiscrete space X is dense in X
4) Every non-empty open subset of an indiscrete space X is not dense in X

22. Let $X = N$ be equipped with the topology generated by the basis consisting of sets $A_n = \{n, n+1, n+2 \ldots\}$, $n \in N$ then X is

1) Compact and connected
2) Hausdorff and compact
3) Hausdorff and connected
4) Neither compact nor connected

23. Every convergent sequence in a Hausdorff space has

1) exactly two different limit points
2) no limit point
3) a unique limit point
4) more than one limit point

24. Let X be a topological space with finitely many connected components: Then each connected components is

1) closed in X
2) open in X
3) neither open nor closed in X
4) both open and closed in X

25. Any infinite subset A of a discrete topological space X is

1) compact

2) locally compact

3) not compact

4) sequentially compact

26. The general solution of $\frac{dy}{dx} = \frac{y}{x} + \frac{1}{\sin\left(\frac{y}{x}\right)}$ is

1) $y = x \; arc \; \cos(x - c)$

2) $y = x \; arc \; \sin(x - c)$

3) $y = x \; arc \; \tan(x - c)$

4) $y = x \; \sin(x - c)$

27. Which of the following is elliptic?

1) Laplace equation

2) Wave equation

3) Heat equation

4) $u_{xx} + 2u_{xy} - 4u_{xy} = 0$

28. The complete integral of the PDE $pq = 1$ is

1) $z = ax + 1/a\, y + c$

2) $z = ax + 1/a\, y$

3) $z = ax + y + c$

4) $z = x + 1/a\, y + c$

29. The PDE $(1+x^2)u_{xx} + (1+y^2)u_{yy} + xu_x + yu_y = 0$ is of type

1) Parabolic

2) Elliptic

3) Hyperbolic

4) Laplace

30. The PDE $x^2(y-1)z_{xx} - x(y^2-1)z_{xy} + y(y^2-1)z_{yy} + z_x = 0$ is hyperbolic in the entire xy-plane except along

1) x-axis

2) y-axis

3) a line parallel to y axis

4) a line parallel to x axis

31. In Newton-Cotes formula, if $f(x)$ is interpolated at equally spaced nodes by a polynomial of degree one, then it represents ——————.

1) Trapezoidal rule

2) Simpson rule

3) Three-eight rule

4) Booles rule

32. By Newton's method $f(x) = x^5 - x^3 + 3$ and if $x_n = 1$ then x_{n+1} is

1) $-\frac{1}{2}$

2) $\frac{1}{2}$

3) $\frac{3}{2}$

4) $-\frac{3}{2}$

33. For the fastest rate of convergence of the method $x_{n-1} = \frac{ax_n - x_n^2 + 1}{a + x_n}$. The value of a is (Given α is exact root)

1) $a = \frac{1}{\alpha^2}$

2) $a = \frac{2}{\alpha^3}$

3) $a = \frac{1}{\alpha^3}$

4) $a = \frac{3}{\alpha^2}$

34. If Euler's characteristic equation $\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y}\right) = 0$ vanishes identically, then the indefinite integral $\int F(x, y, y')\, dx$ can be evaluated as a function of

1) x only

2) x and y

3) y only

4) F, x and y

35. The integral I has strong minimum if

1) The arc AB of the arc of the integration Γ_e, contains no point conjugate to either A or B

2) The arc AB of the arc of integration Γ_e, contains point conjugate to either A or B

3) The arc AB of the arc of integration Γ_e, contains point conjugate to both A or B

4) The arc AB of the arc of the integration Γ_e, contains no point conjugate to neither A nor B

36. The solution of Fredholm integral equation $y(x) = x + e^x - \int_0^1 x\,t\,y(t)\,dt$

1) $y(x) = e^x$

2) $y(x) = e^{-x}$

3) $y(x) = x$

4) $y(x) = x + 1$

37. Consider the Fredholm integral equation of the first kind $\frac{1}{4}e^x = \int_0^{\frac{1}{4}} e^{x-t}\,y(t)\,dt$. The solution is

1) $y(x) = e^x$

2) $y(x) = \cos x$

3) $y(x) = \sin x$

4) $y(x) = e^{-x}$

38. Any solution of homogeneous Volterra integral equation of the second kind $\phi(x) - \lambda \int_0^x K(x \cdot y)\phi(x)\,dy = 0$ in L_2-space is

1) Necessarily a zero function

2) Necessarily a non-zero function

3) Absolute function

4) Constant function

39. The eigen value I of the following Fredholm integral equation $y(x) = I\int_0^1 x^2 t\,y(t)\,dt$ is

1) -2

2) 2

3) 4

4) -4

40. Degree of freedom is defined as

1) The minimum number of independent coordinates required to specify the system

2) The maximum number of independent coordinates required to specify the system

3) The minimum number of dependent coordinates required to specify the system

4) The maximum number of dependent coordinates required to specify the system

41. Non-holonomic constraints are

1) The constraints that can be expressed as equation form

2) The constraints that cannot be expressed as equation form

3) Equation of constraints that contain time as explicit variable

4) Equation of constraints that does not contain time as explicit variable

42. Lagrange's bracket is

1) Canonical invariant

2) Canonical variant

3) Non-invariant

4) Euler's invariant

43. Two dice are thrown. Find the probability that the total of the numbers on the top face is 9.

1) $\frac{3}{9}$

2) $\frac{4}{9}$

3) $\frac{5}{36}$

4) $\frac{4}{36}$

44. From a pack of 52 cards, one card is drawn at random. Find the probability of getting a queen.

1) $\frac{1}{13}$

2) $\frac{2}{13}$

3) $\frac{10}{21}$

4) $\frac{12}{21}$

45. Poisson distribution is a limiting case of

1) Uniform distribution

2) Exponential distribution

3) Geometric distribution

4) Binomial distribution

46. If X is uniformly distributed over $(0, 10)$ find $P(X < 2)$

1) $\frac{3}{5}$

2) $\frac{2}{5}$

3) $\frac{1}{5}$

4) $\frac{4}{5}$

47. If the one-step transition probability does not depend on the step (ie.,) $p_{ij}(n-1, n) = p_{ij}(m-1, m)$ the Markov chain is called a

1) Non-homogeneous

2) Homogeneous

3) Irreducible

4) Reducible

48. A non-null persistent and a periodic state is called

1) Regular

2) Irregular

3) Ergodic

4) Non-Ergodic

49. The variance of maximum likelihood estimate for the parameter λ of a Poisson distribution on the basis of a Poisson distribution on the basis of a sample of size n is

1) λ

2) $\frac{n}{\lambda}$

3) $\frac{\lambda}{n}$

4) $\frac{\lambda}{n^2}$

50. Buses arrive for cleaning at a central depot in groups of five every hour on the hour. The buses are serviced in random order, one at a time. Each bus requires 11 min to service completely and it leaves the depot as soon as it is clean. Then the average number of buses in the depot is

1) 2

2) 2.5

3) 2.75

4) 3

ROUGH WORK

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