Indian Statistical Institute

Junior Research Fellowship in Geology, Entrance Examination

2016

TEST CODE: **GEA**

BOOKLET No.

Forenoon Time: 2 hours

Part I - ten questions **All questions carry 4 marks**
Part II - twenty questions

Give your answers in the answer booklet only.

Write your Name, Registration Number, Test Centre, Test Code and the Number of this booklet in the appropriate places on the answer sheet.

STAPLE/ATTACH QUESTION BOOKLET WITH THE ANSWER BOOKLET. ALL ROUGH WORK MUST BE DONE ON THE QUESTION BOOKLET AND/OR ON THE ANSWER BOOKLET. YOU ARE NOT ALLOWED TO USE CALCULATOR.

WAIT FOR THE SIGNAL TO START WRITING

Part – I

Select the right answer from the given alternatives for each of the following questions. Each question carries 4 marks

1. The locus of the foot of the perpendicular drawn from either focus upon any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is
   (a) $x^2+y^2=a^2$
   (b) $x^2+y^2=b^2$
   (c) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
   (d) $ax+by=1$.
2. If $\alpha=\frac{1}{2}\left(-1+\sqrt{-3}\right)$ and $\beta=\frac{1}{2}\left(-1-\sqrt{-3}\right)$, the value of $\alpha^4+\alpha^2\beta^2+\beta^4$ is
   (a) 0
   (b) 1
   (c) 3
   (d) $\frac{1}{16}$.
3. $\lim_{x\to 0}\frac{xe^x-\log(1+x)}{x^2}$ is equal to
   (a) $\frac{3}{2}$
   (b) $\frac{5}{2}$
   (c) $\frac{1}{2}$
   (d) $\frac{7}{2}$.

4 A function $f(x)$ is defined by the equations

$$f(x) = 1 + x \qquad x \leq 0$$
$$= x \qquad 0 < x < 1$$
$$= 2 - x \qquad 1 \leq x < 2$$
$$= 3x - x^2 \qquad x \geq 2$$

The function $f(x)$ is

(a) continuous at x=0 and x=2
(b) discontinuous at x=0 and x=2
(c) differentiable at x=0 and x=2
(d) none of the above.

5. If $\sin y = x \sin(a + y)$, the value of $\frac{dy}{dx}$ is

(a) $\frac{sin^2(a+y)}{\sin a}$

(b) $\frac{sin^2(a+x)}{\sin a}$

(c) $\frac{cos^2(a+y)}{\cos a}$

(d) $\frac{cos^2(a+x)}{\cos a}$.

6. The maximum value of the function $y = x^{\frac{1}{x}}$ is

(a) $e^{\frac{1}{e}}$

(b) $\left(\frac{1}{e}\right)^e$

(c) e^e

(d) none of the above.

7. $\int_0^{\frac{\pi}{2}} \log \cos x \, dx$ is equal to

(a) $\frac{\pi}{2} \log \frac{1}{2}$

(b) $\frac{\pi}{8} \log 2$

(c) $\pi \log 2$

(d) $\frac{\pi}{8} \log \frac{1}{2}$.

8. The area of the region bounded by the circle $x^2 + y^2 = 25$, the x-axis and the ordinates $x = -3$ and $x = 4$ is

(a) $12 + \frac{25}{2}\frac{\pi}{2}$

(b) $13 + \frac{23}{2}\frac{\pi}{4}$

(c) $14 + \frac{21}{2}\frac{\pi}{6}$

$(d) 15 + \frac{19}{2}\frac{\pi}{3}$.

9. The solution of the differential equation

$$\frac{dy}{dx} = \frac{xy^2 - x}{yx^2 + y}$$

is

(a) $\frac{1}{2}\log(y^2 - 1) = \frac{1}{2}\log(x^2 + 1) + c$

(b) $\frac{1}{2}\log(y - 1) = \frac{1}{2}\log(x + 1) + c$

(c) $\frac{1}{2}\log(y^2+1) = \frac{1}{2}\log(x^2-1) + c$

(d) $\frac{1}{2}\log(y+1) = \frac{1}{2}\log(x-1) + c.$

10. The inverse of the matrix

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$$

is

(a) $\begin{bmatrix} \frac{1}{10} & \frac{3}{10} & \frac{1}{5} \\ \frac{21}{20} & \frac{-7}{20} & \frac{-2}{5} \\ \frac{-9}{10} & \frac{3}{10} & \frac{1}{5} \end{bmatrix}$

(b) $\begin{bmatrix} \frac{2}{20} & \frac{21}{20} & \frac{-18}{20} \\ \frac{6}{20} & \frac{-7}{20} & \frac{6}{20} \\ \frac{1}{5} & \frac{8}{20} & \frac{1}{5} \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 3 & 0 \\ 0 & 4 & -6 \\ -1 & 5 & 7 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 6 & 4 \\ 21 & -7 & -8 \\ -18 & 6 & 4 \end{bmatrix}.$

Part – II

Select the right answer from the given alternatives for each of the following questions. Each question carries 4 marks

11. A circle of radius *a* is deformed into an ellipse. If the transformation matrix is given by $\begin{pmatrix} \sqrt{\lambda 1} & 0 \\ 0 & \sqrt{\lambda 2} \end{pmatrix}$, the strain ellipse axial ratio is given by

A. $\frac{\sqrt{\lambda 1}}{a}$

B. $\frac{\sqrt{\lambda 2}}{a}$

C. $\frac{\lambda 1}{\lambda 2}$

D. $\frac{\sqrt{\lambda 1}}{\sqrt{\lambda 2}}$.

12. Mohr circle for stress is given by the equation

$$\sigma^2 - \sigma\tau(\sigma 1 + \sigma 3) + \tau^2 = -\sigma 1 \sigma 3,$$

where σ & τ stand respectively for normal stress and shear stress, and σ1 & σ3 are the maximum and minimum principal stresses. According to this formulation, when τ is maximum normal stress is given by

A. $(\sigma 1 + \sigma 3)$

B. $(\sigma 1 + \sigma 3)/2$

C. $(\sigma 1 - \sigma 3)$

D. $(\sigma 1 - \sigma 3)/2.$

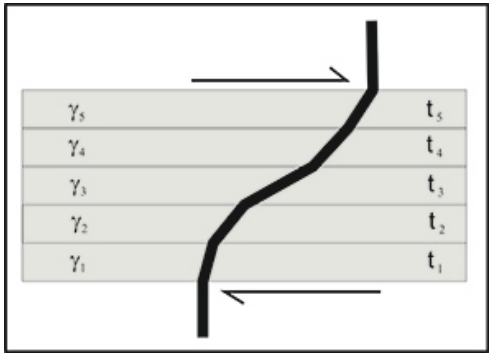
13. If the profile a symmetric recumbent fold is given by

$$y^2 = 4ax,$$

where (0,0) point is the hinge point of the fold and x-axis follows the axial trace, then the profile of a symmetric upright synform with y-axis as axial trace has the form

A. $y^2 = -4ax$
B. $x^2 = 4ay$
C. $y^2 = 2ax$
D. $x^2 = -4ay.$

14. The displacement of a dyke along a ductile shear zone is given schematically by the adjoining diagram, where γ_i represents shear strain in each domain and t_i is the thickness of the corresponding domain.



Total shear displacement along the shear zone is given by

A. $\sum_{i=2}^{i=4} \gamma_i t_i$

B. $\gamma_3 t_3$

C. $\sum_{i=1}^{i=5} \gamma_i t_i$

D. ∞.

15. Erosion rates are considered to be proportional to relief, as expressed by the following equation $\frac{dh}{dt} = -\alpha h$, where h is the relief, dh/dt is the erosion rate, t is the time and α is the erosion constant. The differential equation can be simplified as

A. $h(t) = h_0 \exp(\alpha t)$

B. $h(t) = h_0 \exp(-\alpha t)$

C. $h(t) = h_0 (-\alpha t)$

D. $h(t) = h_0 \log(-\alpha t)$.

16. Calculate the concentration of an element (C) after 50% crystallization (i.e. F = 0.5) if it's initial concentration in the melt (i.e. C_0) was 200.0 ppm and distribution coefficient D = 2.0

A. 200.0

B. 150.0

C. 100.0

D. 250.0.

17. Radioactive minerals become less radioactive (in counts per second) with time according to the equation given below (a is the radioactivity after time t, a_0 in the initial radioactivity, λ is a constant which depends upon the mineral, ln is natural logarithm with base e)

A. $\ln(a) = \ln(a_0) - \lambda t$

B. $\ln(a) = \ln(a_0) + \lambda t$

C. $\ln(a_0) = \ln(a) - \lambda t$

D. $\ln(a_0) = \ln(a) + \lambda t$.

18. Assuming that grains of different sizes have identical densities and are nearly spherical, using Stoke's law the ratio of the settling velocities for two different grain sizes is (where v_1 and v_2 are the velocities for grains of radius r_1 and r_2 respectively; g = gravitational constant, η = fluid viscosity, ρ_p = density of grain, ρ_f = density of fluid)

A. $\frac{v_1}{v_2} = \left(\frac{r_1}{r_2}\right)$

B. $\frac{v_1}{v_2} = \left(\frac{gr_1}{\eta r_2}\right)$

C. $\frac{v_1}{v_2} = \left(\frac{\left(\rho_p - \rho_f\right) r_1}{\left(\rho_p + \rho_f\right) r_2} \right)^2$

D. $\frac{v_1}{v_2} = \left(\frac{r_1}{r_2} \right)^2$.

19. For a simple vertically traveling wave, the travel time *t* from the surface to the reflector and back is called the Two-Way Time (TWT) and is given by the formula (*d* is the depth of the reflector and *V* is the wave velocity in the rock).

    A. $t = 2d/V$

    B. $t = 1/V$

    C. $t = d/2V$

    D. $t = 1/d$.

20. Dissolved nitrate concentration of 18.0 mg/L is being advected with flowing groundwater at a velocity of 0.331 m/day in an aquifer with a porosity of 0.225. Groundwater from aquifer discharges into a stream. If the aquifer is 1.80 m thick and 123.0 m wide where it discharges into the stream the mass flux of nitrate into the stream would be

    A. 297 gm/day

    B. 300 gm/day

    C. 290 gm/day

D. 295gm/day.

21. We came across 21 measurements of pebble long axis in a notebook. The mean of pebble long axes of a particular river is known to be 5cm and the standard deviation is unknown. The adjusted sample standard deviation is 0.5 cm. We need to know if the 21 measurements are from this river or not. What test statistics should we choose, $\bar{X}$ being sample mean?

A. $\frac{\bar{X}-5}{0.5/\sqrt{20}}$ follows t (with 20 degrees of freedom)

B. $\frac{\bar{X}-5}{0.5/\sqrt{21}}$ follows t (with 20 degrees of freedom)

C. $\frac{\bar{X}-5}{0.5/\sqrt{21}}$ follows t (with 21 degrees of freedom)

D. $\frac{\bar{X}-5}{0.5/21}$ follows t (with 20 degrees of freedom).

22. A senior palaeontologist has a very large collection of Unio shells from the Triassic beds of the Rewa Gondwana Basin. Another geologist has recently collected 15 Unio shells from the Triassic beds of the Satpura Basin which appear generally larger than the collection of the senior. They wanted to see if the population of the new collection (μ_1) has a mean equal to that of the senior's collection (μ_0). A 't-test' was performed. The value of the sample statistic has been found to be less than the critical value at 5% level of confidence. What will they conclude?

A. Conclude that $\mu_1 \neq \mu_0$ with the possibility of type-I error
B. Conclude that $\mu_1 = \mu_0$ with the possibility of type-I error
C. Conclude that $\mu_1 \neq \mu_0$ with the possibility of type-II error
D. Conclude that $\mu_1 = \mu_0$ with the possibility of type-II error.

23. A dataset comprises the length of pebble long axes reported in centimetre. If the standard deviation of that data is computed using standard methodology then in what unit the standard deviation will be reported?

A. Unit less
B. Centimetre
C. Centimeter^2
D. Percentage.

24. If a rectangular block of rock, having sides 5, 7 and 9 cm, has been reduced to an ellipsoid having long axes 1, 3 and 7 cm due to abrasion then what is the approximate volume of rock that has been removed?

A. 227 cm^3
B. 294 cm^3
C. 315 cm^2
D. 88 cm^3.

25. If three seismographs are stationed at three vertices of a triangle then they will pick-up the primary wave simultaneously if the earthquake takes place at the

    A. Circumcentre
    B. Incentre
    C. Orthocentre
    D. Centroid.

26. For a spherical earth what will be the ratio of the perimeter lengths of the 60^{o} latitude and the equator?

    A. 1.5
    B. 2
    C. 0.5
    D. 3 .

27. Five grains of sand are being transported through continuous saltation in water. Their movement is such that they come in contact with the substrate every 2, 3, 4, 7, and 9 seconds respectively. If they were lifted simultaneously after how many seconds will they touch the substrate together?

    A. 42
    B. 1512
    C. 252
    D. 9 .

28. For an exfoliated granite hill which is *h* meter high, *x* is the map distance away from the peak along a straight line and *z* is the altitude above the mean sea level. The cross sectional profile (*x*, *z* plane) for one side of the hill could be best approximated by:

A. $z = h - x^2$

B. $z = \frac{h}{x}$

C. $z = h - \sqrt{x}$

D. $z = h \times x.$

29. Length of some trilobite shells are

No.	Length (mm)
1	205
2	255
3	195
4	220
5	235

The standard deviation of their length is

A. 23.88

B. 45.88
C. 570
D. 1110.

30. In a normal distribution, slightly over 2/3 that is 68.27% of observations fall within

A. 1 standard deviation on either side of the mean
B. 2 standard deviation on either side of the mean
C. 3 standard deviation on either side of the mean
D. 4 standard deviation on either side of the mean.