

Indian Statistical Institute

Junior Research Fellowship in Geology, Entrance Examination 2014

BOOKLET No.

TEST CODE: **GEA**

Forenoon

Time: 2 hours

Part I - ten questions

10 X 4 = 40

Part II - six questions

6 X 10 = 60

Total

100

Give your answers in the answer booklet only.

Write your Name, Registration Number, Test Centre, Test Code and the Number of this booklet in the appropriate places on the answer sheet.

STAPLE/ATTACH QUESTION BOOKLET WITH THE ANSWER BOOKLET. ALL ROUGH WORK MUST BE DONE ON THE QUESTION BOOKLET AND / OR ON THE ANSWER BOOKLET. YOU ARE NOT ALLOWED TO USE CALCULATOR.

WAIT FOR THE SIGNAL TO START WRITING

Part-I

Select the right answer from the given alternatives for each of the following questions.

10×4=40

- The locus of the middle points of all chords of the parabola $y^2 = 8x$, which pass through the point (1,-2) is
 - $y^2 + 2y = 4(x - 1)$
 - $x^2 + 2x = 4(y - 1)$
 - $x^2 + 4y = 2(y - 1)$
 - None of these
- The value of $\tan \left\{ i \log \frac{a-ib}{a+ib} \right\}$ is
 - $\frac{2ab}{a^2-b^2}$
 - $\frac{2ab}{a^2+b^2}$
 - $\frac{a^2+b^2}{2ab}$
 - None of these
- $\lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\tan x}$ is equal to
 - 1
 - 0
 - 2
 - None of these
- If $\tan y = \frac{2t}{1-t^2}$, $\sin x = \frac{2t}{1+t^2}$, then $\frac{dy}{dx}$ is
 - 1
 - 2
 - 3
 - None of these
- If $y = a(1 - \cos \theta)$, $x = a(\theta - \sin \theta)$, y being regarded as a function of x , the function is maximum at

- (a) $\theta = \pi$
- (b) $\theta = 0$
- (c) $\theta = \frac{\pi}{2}$
- (d) None of these

6. The integral $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$ is equal to

- (a) $\frac{\pi^2}{4}$
- (b) $\frac{\pi^2}{2}$
- (c) $\frac{\pi}{8}$
- (d) None of these

7. The solution of the differential equation

$$\frac{dy}{dx} + \frac{y}{x} = y^2$$

is

- (a) $\frac{1}{y} = cx - x \log x$
- (b) $x + y = cxy$
- (c) $\log xy + (x - y) = c$
- (d) None of these

8. The inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$$

is

- (a) $\begin{bmatrix} \frac{1}{10} & \frac{8}{10} & \frac{1}{5} \\ \frac{21}{20} & \frac{-7}{20} & \frac{-2}{5} \\ \frac{-9}{10} & \frac{8}{10} & \frac{1}{5} \end{bmatrix}$
- (b) $\begin{bmatrix} 1 & 8 & 0 \\ 0 & 4 & -6 \\ -1 & 5 & -7 \end{bmatrix}$
- (c) $\begin{bmatrix} 2 & 6 & 4 \\ 21 & -7 & -8 \\ -18 & 6 & 4 \end{bmatrix}$
- (d) None of these

9. The characteristic roots and the corresponding characteristic vectors of the matrix

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 8 & -4 & 3 \end{bmatrix}.$$

are

- (a) $0, (1 \ 2 \ 2)'; 3, (2 \ 1 \ -2)'; 15, (2 \ 2 \ 1)'$
 (b) $1, (2 \ 3 \ 3)'; 2, (1 \ 3 \ -3)'; 8, (3 \ 3 \ 2)'$
 (c) $4, (3 \ 2 \ 2)'; 5, (4 \ 2 \ -6)'; 10, (1 \ 2 \ 2)'$
 (d) None of these

10. The values of θ for which the equations

$$\begin{aligned} x + y + z &= 1 \\ x + 2y + 4z &= \theta \\ x + 4y + 10z &= \theta^2 \end{aligned}$$

are consistent and the corresponding solutions are

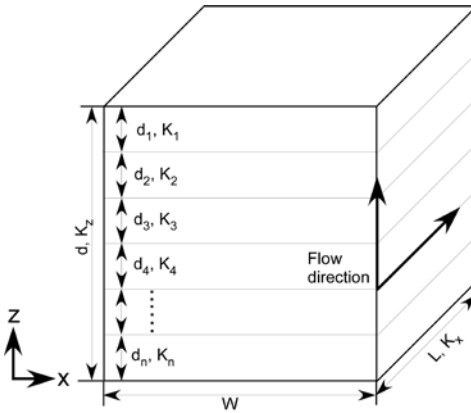
- (a) $1, x=-1, y=3, z=-1; 2, x=-2, y=4, z=-1$
 (b) $3, x=1, y=2, z=-2$
 (c) $4, x=2, y=3, z=-4$
 (d) None of these

Part-II

(Six questions, ten marks each)

11. Consider the layered formation shown in the figure below. Each layer (1, 2, 3, 4, ..., n) is homogeneous and isotropic with hydraulic conductivity values $K_1, K_2, K_3, K_4, \dots, K_n$, but the system as a whole will act like a single homogeneous, anisotropic layer. Now consider flow perpendicular and parallel to the layering. Assume flux Q entering and leaving the system is constant throughout ($Q =$

$KA(dh/dl)$). Show that $K_z = \frac{d}{\sum_{i=1}^n \frac{d_i}{K_i}}$ and $K_x = \frac{\sum_{i=1}^n d_i K_i}{d}$.



12. A coal seam is encountered at boreholes A, B and C at depths of 100m, 200m and 200m respectively.

- (a) B and C are located 100m to the East and 100m to the North respectively of A. The strike and dip of the coal seam is best given by
- 130, 40° NE.
 - 135, 45° NE.
 - 120, 45° NE.
 - 135, 55° NE.
- (b) A fourth borehole is drilled at D, 100m East of B. Find the depth at which the coal seam will be encountered at D.
- (c) Which of the following sets of assumptions is necessary in solving the problem above?
- All the boreholes are on a horizontal ground and the boreholes are vertical.
 - Surface elevations at which the boreholes are drilled are the same and the boreholes are vertical.
 - Surface elevations at which the boreholes are drilled are the same, the coal seam has uniform dip and the boreholes are vertical.

4+3+3

13. Following table provides the chemical compositions of ten analyzed mineral grains as oxide weight percent with an analytical error of $\pm 2\%$.

Analysis #	Na ₂ O	MgO	SiO ₂	Al ₂ O ₃	K ₂ O	CaO	TiO ₂	Cr ₂ O ₃	MnO	FeO	P ₂ O ₅	Total
1	0.21	0.03	63.64	18.61	16.54	0.02	0.09	0.07	0.04	0.08	0	99.32
2	0.18	0.02	63.4	18.32	17.48	0	0	0	0	0.07	0	99.46
3	0.09	0	64.36	18.6	17.41	0	0.14	0	0	0	0	100.61
4	0.12	0.02	63.71	18.18	16.96	0	0.01	0.04	0	0.26	0	99.31
5	0.07	0.05	0.31	0	0.06	56.63	0	0	0	0.79	42.39	100.3
6	0.03	0.07	0.27	0.02	0.04	55.87	0	0	0.15	0.71	42.23	99.37
7	0.33	0.08	62.56	19.44	16.61	0	0.07	0	0	0.27	0	99.37
8	0.18	0	64.5	18.58	16.85	0	0.06	0.01	0.05	0.1	0.02	100.36
9	0.18	0.16	63.64	18.66	16.93	0	0.06	0	0	0.44	0	100.08
10	0.03	0	99.49	0.17	0.02	0	0	0	0.07	0.04	0	99.81

- (a) How many distinct mineral species do the analyses represent?
 (b) How many of these are non-silicates? Identify the analysis serial number(s) in such case(s)?
 (c) One of the mineral species is feldspar. Which of the following equations would give the best estimate of average value of K₂O in feldspar?

i)
$$\frac{\sum_{i=1}^{10} (K_2O)_i}{10}$$

ii)
$$\frac{\sum_{i=1}^4 (K_2O)_i}{4} + \frac{\sum_{i=7}^9 (K_2O)_i}{3}$$

iii)
$$\frac{\left(\frac{\sum_{i=1}^4 (K_2O)_i}{4} + \frac{\sum_{i=7}^9 (K_2O)_i}{3} \right)}{2}$$

iv)
$$\frac{\sum_{i=7}^9 (K_2O)_i}{3}$$

- (d) An estimate of the mean and standard deviation of an oxide percent is given as 18.627 ± 0.399 . Which of the following oxides is correctly associated with this estimate?

- i) Al₂O₃
 ii) SiO₂
 iii) CaO
 iv) K₂O

Q14. A systematist is exploring a planet in another solar system and discovers the following four different species, A, B, C and D.

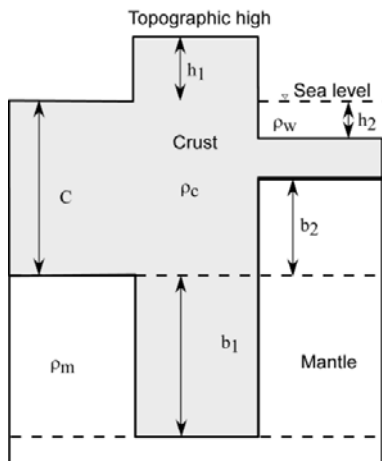
The character matrix generated is as follows with '0' as primitive and '1' as derived character states

Character	A	B	C	D
1. "Feet" present	0	0	0	0
2. Spines present	0	1	0	0
3. Eyes on stalks	0	1	1	1
4. "Tail" present	0	0	1	1
5. Antennae present	0	0	0	1

Draw a cladogram from the given matrix. If you draw a dendrogram using the above matrix, will that be a valid taxonomic tree? Justify your answer. 6+4

15. Carefully study the figure given below. Explain compensation depth using the figure below. If the density of the mantle (ρ_m) is assumed to be $3,300 \text{ kg m}^{-3}$, density of the crust (ρ_c) is assumed to be $2,750 \text{ kg m}^{-3}$ and density of water (ρ_w) is assumed to be 1000 kg m^{-3} then show that $b_1 \approx 5h_1$ and, $b_2 \approx 3.2h_w$.

2+4+4



16. A person had measured the length of long axis of 20 pebbles and had found the mean length to be 2.4 cm. He tabulated his observations as follows:

Length in cm	Number of observations
0.0-1.0	2
1.0-2.0	-
2.0-3.0	8
3.0-4.0	-
4.0-5.0	2

The number of observation noted for the two class means (i.e., 1.0-2.0 and 3.0-4.0cm) got smudged beyond legibility as his note book fell into the river. Could you help him to reconstruct the table and find the median length of the pebble long axis?

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