

2016

BOOKLET NO.

TEST CODE : **MMA**

*Forenoon*

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| <b>Questions : 30</b> | <b>Time : 2 hours</b> |
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*Write your Name, Registration Number, Test Centre, Test Code and the Number of this booklet in the appropriate places on the answersheet.*

For each question, there are four suggested answers of which only one is correct. For each question indicate your choice of the correct answer by darkening the appropriate oval (●) completely on the answer sheet.

4 marks are allotted for each correct answer,  
0 mark for each incorrect answer and  
1 mark for each unattempted question.

ALL ROUGH WORK MUST BE DONE ON THIS BOOKLET ONLY.  
YOU ARE NOT ALLOWED TO USE CALCULATORS IN ANY FORM.

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**STOP! WAIT FOR THE SIGNAL TO START.**

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MMA<sub>e</sub>-1



1. Suppose  $a, b, c > 0$  are in geometric progression and  $a^p = b^q = c^r \neq 1$ . Which one of the following is always true?

(A)  $p, q, r$  are in geometric progression  
 (B)  $p, q, r$  are in arithmetic progression  
 (C)  $p, q, r$  are in harmonic progression  
 (D)  $p = q = r$

2. How many complex numbers  $z$  are there such that  $|z + 1| = |z + i|$  and  $|z| = 5$ ?

(A) 0 (B) 1 (C) 2 (D) 3

3. The number of real roots of the equation

$$2 \cos \left( \frac{x^2 + x}{6} \right) = 2^x + 2^{-x} \quad \text{is}$$

(A) 0 (B) 1 (C) 2 (D)  $\infty$

4. If  $a, b, c$  and  $d$  satisfy the equations

$$\begin{aligned} a + 7b + 3c + 5d &= 16 \\ 8a + 4b + 6c + 2d &= -16 \\ 2a + 6b + 4c + 8d &= 16 \\ 5a + 3b + 7c + d &= -16 \end{aligned}$$

Then  $(a + d)(b + c)$  equals

(A)  $-4$  (B) 0 (C) 16 (D)  $-16$

5. Let

$$f(x, y) = \begin{cases} \frac{x^2 y}{x^4 + y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

Then  $\lim_{(x, y) \rightarrow (0, 0)} f(x, y)$

(A) equals 0 (B) equals 1 (C) equals 2 (D) does not exist

6. Find the centroid of the triangle whose sides are given by the following equations:

$$\begin{aligned} 4x - y &= 19 \\ x - y &= 4 \\ x + 2y &= -11 \end{aligned}$$

(A)  $\left(\frac{11}{3}, -\frac{7}{3}\right)$  (B)  $\left(\frac{5}{3}, -\frac{7}{3}\right)$  (C)  $\left(-\frac{11}{3}, -\frac{7}{3}\right)$  (D)  $\left(\frac{7}{3}, -\frac{11}{3}\right)$

7. The set of value(s) of  $\alpha$  for which  $y(t) = t^\alpha$  is a solution to the differential equation

$$t^2 \frac{d^2 y}{dx^2} - 2t \frac{dy}{dx} + 2y = 0 \quad \text{for } t > 0 \quad \text{is}$$

(A)  $\{1\}$  (B)  $\{1, -1\}$  (C)  $\{1, 2\}$  (D)  $\{-1, 2\}$

8. Let  $g : \mathbb{R} \rightarrow \mathbb{R}$  be differentiable with  $g'(x^2) = x^3$  for all  $x > 0$  and  $g(1) = 1$ . Then  $g(4)$  equals

(A)  $64/5$  (B)  $32/5$  (C)  $37/5$  (D)  $67/5$

9. Suppose  $X$  and  $Y$  are two independent random variables both following Poisson distribution with parameter  $\lambda$ . What is the value of  $E(X - Y)^2$ ?

(A)  $\lambda$  (B)  $2\lambda$  (C)  $\lambda^2$  (D)  $4\lambda^2$

10. If  $A_1, A_2, \dots, A_n$  are independent events with probabilities  $p_1, p_2, \dots, p_n$  respectively, then

$$P\left(\bigcup_{i=1}^n A_i\right)$$

equals

(A)  $\sum_{i=1}^n p_i$  (B)  $\prod_{i=1}^n p_i$  (C)  $\prod_{i=1}^n (1 - p_i)$  (D)  $1 - \prod_{i=1}^n (1 - p_i)$

11. Ravi asked his neighbor to water a delicate plant while he is away. Without water, the plant would die with probability  $\frac{4}{5}$  and with water it would die with probability  $\frac{3}{20}$ . The probability that Ravi's neighbor would remember to water the plant is  $\frac{9}{10}$ . If the plant actually died, what is the probability that Ravi's neighbor forgot to water the plant?
- (A)  $\frac{4}{5}$                       (B)  $\frac{27}{43}$                       (C)  $\frac{16}{43}$                       (D)  $\frac{2}{25}$
12. Suppose there are  $n$  positive real numbers such that their sum is 20 and the product is strictly greater than 1. What is the maximum possible value of  $n$ ?
- (A) 18                      (B) 19                      (C) 20                      (D) 21
13. Which one of the following statements is correct regarding the elements and subsets of the set  $\{1, 2, \{1, 2, 3\}\}$ ?
- (A)  $\{1, 2\} \in \{1, 2, \{1, 2, 3\}\}$                       (B)  $\{1, 2\} \subseteq \{1, 2, \{1, 2, 3\}\}$   
 (C)  $\{1, 2, 3\} \subseteq \{1, 2, \{1, 2, 3\}\}$                       (D)  $3 \in \{1, 2, \{1, 2, 3\}\}$
14. The number of terms independent of  $x$  in the binomial expansion of  $\left(3x^2 + \frac{1}{x}\right)^{10}$  is
- (A) 0                      (B) 1                      (C) 2                      (D) 5
15. The number of positive integers  $n$  for which  $n^2 + 96$  is a perfect square is
- (A) 0                      (B) 1                      (C) 2                      (D) 4
16. Suppose a 6 digit number  $N$  is formed by rearranging the digits of the number 123456. If  $N$  is divisible by 5, then the set of all possible remainders when  $N$  is divided by 45 is
- (A)  $\{30\}$                       (B)  $\{15, 30\}$                       (C)  $\{0, 15, 30\}$                       (D)  $\{0, 5, 15, 30\}$

17. The number of positive integers  $n$  for which

$$n^3 + (n+1)^3 + (n+2)^3 = (n+3)^3$$

is

- (A) 0                      (B) 1                      (C) 2                      (D) 3

18. Let  $A = \begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix}$ , and  $B = A + A^2 + A^3 + \cdots + A^{50}$ . Then

- (A)  $B^2 = I$               (B)  $B^2 = 0$               (C)  $B^2 = A$               (D)  $B^2 = B$

19. Let  $A$  be a real  $2 \times 2$  matrix. If  $5 + 3i$  is an eigenvalue of  $A$ , then  $\det(A)$

- (A) equals 4                      (B) equals 8                      (C) equals 16  
(D) cannot be determined from the given information

20. Let  $f : (0, \infty) \rightarrow (0, \infty)$  be a strictly decreasing function. Consider

$$h(x) = \frac{f\left(\frac{x}{1+x}\right)}{1 + f\left(\frac{x}{1+x}\right)}.$$

Which one of the following is always true?

- (A)  $h$  is strictly decreasing  
(B)  $h$  is strictly increasing  
(C)  $h$  is strictly decreasing at first and then strictly increasing  
(D)  $h$  is strictly increasing at first and then strictly decreasing

21. Let  $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ . How many functions  $f : A \rightarrow A$  can be defined such that  $f(1) < f(2) < f(3)$ ?

- (A)  $\binom{8}{3}$                       (B)  $\binom{8}{3} 5^8$                       (C)  $\binom{8}{3} 8^5$                       (D)  $\frac{8!}{3!}$

22. The infinite series  $\sum_{n=1}^{\infty} \frac{a^n \log n}{n^2}$  converges if and only if
- (A)  $a \in [-1, 1)$  (B)  $a \in (-1, 1]$  (C)  $a \in [-1, 1]$  (D)  $a \in (-\infty, \infty)$
23. Given that  $\int_{-\infty}^{\infty} e^{-x^2/2} dx = \sqrt{2\pi}$ , what is the value of
- $$\int_{-\infty}^{\infty} |x|^{-1/2} e^{-|x|} dx?$$
- (A) 0 (B)  $\sqrt{\pi}$  (C)  $2\sqrt{\pi}$  (D)  $\infty$
24. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a strictly increasing function. Then which one of the following is always true?
- (A) The limits  $\lim_{x \rightarrow a+} f(x)$  and  $\lim_{x \rightarrow a-} f(x)$  exist for all real numbers  $a$
- (B) If  $f$  is differentiable at  $a$  then  $f'(a) > 0$
- (C) There cannot be any real number  $B$  such that  $f(x) < B$  for all real  $x$
- (D) There cannot be any real number  $L$  such that  $f(x) > L$  for all real  $x$
25. An integer is said to be a *palindrome* if it reads the same forward or backward. For example, the integer 14541 is a 5-digit palindrome and 12345 is not a palindrome.
- How many 8 digit palindromes are prime?
- (A) 0 (B) 1 (C) 11 (D) 19
26. Let  $x$  and  $y$  be real numbers satisfying  $9x^2 + 16y^2 = 1$ . Then  $(x + y)$  is maximum when
- (A)  $y = 9x/16$  (B)  $y = -9x/16$  (C)  $y = 4x/3$  (D)  $y = -4x/3$

27. Consider the function

$$f(x) = \frac{e^{-|x|}}{\max\{e^x, e^{-x}\}}, \quad x \in \mathbb{R}.$$

Then

- (A)  $f$  is not continuous at some points
  - (B)  $f$  is continuous everywhere, but not differentiable anywhere
  - (C)  $f$  is continuous everywhere, but not differentiable at exactly one point
  - (D)  $f$  is differentiable everywhere
28. Let  $A$  be a square matrix such that  $A^3 = 0$ , but  $A^2 \neq 0$ . Then which of the following statements is not necessarily true?
- (A)  $A \neq A^2$
  - (B) Eigenvalues of  $A^2$  are all zero
  - (C)  $\text{rank}(A) > \text{rank}(A^2)$
  - (D)  $\text{rank}(A) > \text{trace}(A)$
29. Suppose  $a$  is a real number for which all the roots of the equation  $x^4 - 2ax^2 + x + a^2 - a = 0$  are real. Then
- (A)  $a < -\frac{2}{3}$       (B)  $a = 0$       (C)  $0 < a < \frac{3}{4}$       (D)  $a \geq \frac{3}{4}$
30. A club with  $n$  members is organized into four committees so that each member belongs to exactly two committees and each pair of committees has exactly one member in common. Then
- (A)  $n = 4$
  - (B)  $n = 6$
  - (C)  $n = 8$
  - (D)  $n$  cannot be determined from the given information