2015

BOOKLET NO. TEST CODE : **MMA**

Forenoon

Questions : 30	**Time : 2 hours**

Write your Name, Registration Number, Test Centre, Test Code and the Number of this booklet in the appropriate places on the answersheet.

For each question, there are four suggested answers of which only one is correct. For each question indicate your choice of the correct answer by darkening the appropriate oval (⬬) completely on the answer sheet.

4 marks are allotted for each correct answer,
0 mark for each incorrect answer and
1 mark for each unattempted question.

ALL ROUGH WORK MUST BE DONE ON THIS BOOKLET ONLY.
YOU ARE NOT ALLOWED TO USE CALCULATORS IN ANY FORM.

STOP! WAIT FOR THE SIGNAL TO START.

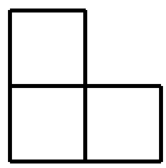
1. A new sequence is obtained from the sequence of positive integers $\{1, 2, 3, \ldots\}$ by deleting all the perfect squares. Then the 2015-th term of the new sequence is C

   (A) 2058 (B) 2059 (C) 2060 (D) 2062.

2. The maximum value of $\cos\alpha_1 \cdot \cos\alpha_2 \cdots \cos\alpha_n$ under the conditions $0 \leq \alpha_i \leq \pi/2$ for all i and $\cot\alpha_1 \cdot \cot\alpha_2 \cdots \cot\alpha_n = 1$ is A

   (A) $\frac{1}{2^{n/2}}$ (B) $\frac{1}{2^n}$ (C) $\frac{1}{2n}$ (D) none of these.

3. Three distinct squares are selected at random from a 8×8 chess board. Then the probability that they form an L-shaped pattern (looked at from one fixed side only) as drawn below is B



   (A) $196/\binom{64}{3}$ (B) $49/\binom{64}{3}$ (C) $36/\binom{64}{3}$ (D) greater than $1/2$.

4. The number of functions $f : \{1, 2, \ldots, 10\} \to \{1, 2, \ldots, 10\}$ such that $f(x) \neq x$ for all x is B

   (A) 10! (B) 9^{10} (C) 10^9 (D) $10^{10} - 1$.

5. The set of all real numbers satisfying $y^2 - 2y - x^2 + 4x = 3$ is a D

   (A) circle (B) point (C) hyperbola (D) pair of straight lines.

6. The fractional part of $\frac{5^{24}}{24}$ equals C

   (A) $5/24$ (B) $9/24$ (C) $1/24$ (D) none of these.

7. Suppose X is distributed as Poisson with mean λ. Then $E(1/(X+1))$ is C

(A) $\dfrac{e^{\lambda}-1}{\lambda}$ (B) $\dfrac{e^{\lambda}-1}{\lambda+1}$ (C) $\dfrac{1-e^{-\lambda}}{\lambda}$ (D) $\dfrac{1-e^{-\lambda}}{\lambda+1}$.

8. In a triangle with sides of length a, b, c, suppose $b+c=x$ and $bc=y$. If also $(x+a)(x-a)=y$, then the triangle is necessarily D

(A) equilateral (B) right angled

(C) acute angled (D) obtuse.

9. Let

$$f(x)=\lim_{n\to\infty}\frac{\log(2+x)-x^{2n}\sin x}{1+x^{2n}} \text{ for } x>0.$$

Then B

(A) f is continuous at $x=1$

(B) $\lim_{x\to 1+} f(x) \neq \lim_{x\to 1-} f(x)$

(C) $\lim_{x\to 1+} f(x)=\sin 1$

(D) $\lim_{x\to 1-} f(x)$ does not exist.

10. Suppose a real matrix A satisfies $A^3=A$, $A\neq I$, $A\neq 0$. If $\mathrm{Rank}(A)=r$ and $\mathrm{Trace}(A)=t$, then B

(A) $r\geq t$ and $r+t$ is odd

(B) $r\geq t$ and $r+t$ is even

(C) $r<t$ and $r+t$ is odd

(D) $r<t$ and $r+t$ is even.

11. The equation $e^x\dfrac{dy}{dx}+3y=x^2y$ is C

(A) separable and not linear

(B) linear and not separable

(C) separable and linear

(D) neither separable nor linear.

12. Let G be the cyclic group generated by an element a of order 30. What is the order of a^{18}? D

(A) 30 (B) 10 (C) 6 (D) none of these.

13. The remainder when $x^{2015} + x^{2014} + 2015$ is divided by $x^2 - 1$ equals A

(A) $x + 2016$ (B) $x - 2016$ (C) $2016x + 1$ (D) $x + 2015$.

14. If P, Q are two invertible matrices such that $PQ = -QP$, then A

(A) $\mathrm{Trace}(P) = \mathrm{Trace}(Q) = 0$

(B) $\mathrm{Trace}(P) = \mathrm{Trace}(Q) = 1$

(C) $\mathrm{Trace}(P) \neq \mathrm{Trace}(Q)$

(D) None of these.

15. Let f be a convex function, i.e.,

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y)$$

for all $0 \leq t \leq 1$ and $x, y \in \mathbb{R}$. Then which of the following is necessarily true? A

(A) $2f(0) + f(4) \geq 2f(1) + f(2)$

(B) fg is a convex function whenever g is convex

(C) f is nondecreasing

(D) none of these.

16. Suppose A is a 100×100 real symmetric matrix whose diagonal entries are all positive. Then which of the following is necessarily true? C

(A) All eigenvalues of A are greater than 0

(B) no eigenvalue of A is greater than 0

(C) at least one eigenvalue of A is greater than 0

(D) none of these.

17. The function $F(k)$ is defined for positive integers as $F(1) = 1,\ F(2) = 1,\ F(3) = -1$ and $F(2k) = F(k),\ F(2k+1) = F(k)$ for $k \geq 2$. Then $F(1) + F(2) + \cdots + F(63)$ equals A

(A) 1 (B) -1 (C) -32 (D) 32.

18. For $a > 0$, the series

$$\sum_{n=2}^{\infty} a^{\log_e n}$$

is convergent if and only if D

(A) $0 < a < 1$ (B) $0 < a \leq e$

(C) $0 < a < e$ (D) $0 < a < 1/e$.

19. Let

$$f(x) = x^2 + \frac{1}{x^2} + x + \frac{1}{x}, \quad x > 0$$

and let $m = \min\{f(x)\}$. Then B

(A) $m = 1$ (B) $m = 4$ (C) $m = 27/4$ (D) m does not exist.

20. The integral

$$\int_0^1 \frac{\sin x}{x^\alpha}\, dx$$

(A) is finite only for $\alpha = 0$ C
(B) is finite only for $|\alpha| < 1$
(C) is finite for all $\alpha < 2$
(D) is infinite for any value of α.

21. Given θ in the range $0 \leq \theta < \pi$, the equation

$$2x^2 + 2y^2 + 4x\cos\theta + 8y\sin\theta + 5 = 0$$

represents a circle for all θ in the interval B

(A) $0 < \theta < \pi/3$ (B) $\pi/4 < \theta < 3\pi/4$

(C) $0 < \theta < \pi/2$ (D) $0 \leq \theta < \pi$.

22. For a natural number n, let $d(n)$ denote the number of divisors of n, including 1 and n. If $1525 \leq n \leq 1675$ and $d(n) = 21$, then n equals B

(A) 1550 (B) 1600 (C) 1625 (D) 1650.

23. How many 5×5 matrices are there such that each entry is 0 or 1 and each row sum and each column sum is 4? C

(A) 64 (B) 32 (C) 120 (D) 96.

24. There are 10 boxes each containing 6 white and 7 red balls. Two random boxes are chosen, one ball is drawn simultaneously at random from each and transferred to the other box. Now a box is again chosen from the 10 boxes and a ball is chosen from it. Then the probability that this ball is white is A

(A) 6/13 (B) 7/13 (C) 5/13 (D) none of these.

25. The integral

$$\int_0^\infty \int_0^\infty \frac{e^{-(x+y)}}{x+y}\, dxdy$$

is C

(A) infinite

(B) finite, but cannot be evaluated in closed form

(C) 1

(D) 2.

26. Let

$$A_n = \frac{1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \cdots \text{ upto n terms}}{n(1 \cdot 2 + 2 \cdot 3 + \cdots \text{ upto n terms})}.$$

Then $\lim_{n\to\infty} A_n$ is C

(A) 1/4 (B) 1/2 (C) 3/4 (D) 5/4.

27. For $n \geq 1$, let G_n be the geometric mean of $\{\sin(\frac{\pi}{2} \cdot \frac{k}{n}) : 1 \leq k \leq n\}$. Then $\lim_{n\to\infty} G_n$ is D

(A) $1/4$ (B) $\log 2$ (C) $\frac{1}{2}\log 2$ (D) $1/2$.

28. Suppose a, b, x, y are real numbers such that $a^2+b^2 = 81$, $x^2+y^2 = 121$ and $ax + by = 99$. Then the set of all possible values of $ay - bx$ is A

(A) $\{0\}$ (B) $\left(0, \frac{9}{11}\right]$ (C) $\left(0, \frac{9}{11}\right)$ (D) $\left[\frac{9}{11}, \infty\right)$.

29. A solution of

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} - 2x = 0$$

that satisfies $x(0) = 3$ and remains bounded as $t \to \infty$ is C

(A) $x = 3e^{-t}$ (B) $x = 4e^{-2t} - e^t$ (C) $x = 3e^{-2t}$ (D) $x = 2e^{-2t} + e^{-t}$.

30. Let $G_1 = \{1, -1, i, -i\}$ and $G_2 = \{1, \omega, \omega^2\}$, where $i = \sqrt{-1}$ and ω is a complex cube root of 1. Define an operation on the Cartesian product $G = G_1 \times G_2$ by

$$(x_1, y_1) \star (x_2, y_2) = (x_1x_2, y_1y_2).$$

Then D

(A) $(G, \star)$ is not a group
(B) $(G, \star)$ is a group but not cyclic
(C) $(G, \star)$ is a group but not commutative
(D) $(G, \star)$ is a commutative cyclic group.