2016

BOOKLET NO. TEST CODE : **MMA**

Forenoon

Questions : 30	**Time : 2 hours**

Write your Name, Registration Number, Test Centre, Test Code and the Number of this booklet in the appropriate places on the answersheet.

For each question, there are four suggested answers of which only one is correct. For each question indicate your choice of the correct answer by darkening the appropriate oval (⬬) completely on the answer sheet.

4 marks are allotted for each correct answer,
0 mark for each incorrect answer and
1 mark for each unattempted question.

ALL ROUGH WORK MUST BE DONE ON THIS BOOKLET ONLY.
YOU ARE NOT ALLOWED TO USE CALCULATORS IN ANY FORM.

STOP! WAIT FOR THE SIGNAL TO START.

1. Suppose $a, b, c > 0$ are in geometric progression and $a^p = b^q = c^r \neq 1$. Which one of the following is always true?

   (A) p, q, r are in geometric progression

   (B) p, q, r are in arithmetic progression

   (C) p, q, r are in harmonic progression

   (D) $p = q = r$

2. How many complex numbers z are there such that $|z+1| = |z+i|$ and $|z| = 5$?

   (A) 0 (B) 1 (C) 2 (D) 3

3. The number of real roots of the equation

$$2\cos\left(\frac{x^2+x}{6}\right) = 2^x + 2^{-x} \quad \text{is}$$

   (A) 0 (B) 1 (C) 2 (D) ∞

4. If a, b, c and d satisfy the equations

$$\begin{array}{rcrcrcrcr} a & + & 7b & + & 3c & + & 5d & = & 16 \\ 8a & + & 4b & + & 6c & + & 2d & = & -16 \\ 2a & + & 6b & + & 4c & + & 8d & = & 16 \\ 5a & + & 3b & + & 7c & + & d & = & -16 \end{array}$$

   Then $(a+d)(b+c)$ equals

   (A) -4 (B) 0 (C) 16 (D) -16

5. Let

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^4+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

   Then $\lim_{(x,y)\to(0,0)} f(x,y)$

   (A) equals 0 (B) equals 1 (C) equals 2 (D) does not exist

6. Find the centroid of the triangle whose sides are given by the following equations:

$$\begin{array}{rcrcr} 4x & - & y & = & 19 \\ x & - & y & = & 4 \\ x & + & 2y & = & -11 \end{array}$$

(A) $\left(\frac{11}{3}, -\frac{7}{3}\right)$ (B) $\left(\frac{5}{3}, -\frac{7}{3}\right)$ (C) $\left(-\frac{11}{3}, -\frac{7}{3}\right)$ (D) $\left(\frac{7}{3}, -\frac{11}{3}\right)$

7. The set of value(s) of α for which $y(t) = t^\alpha$ is a solution to the differential equation

$$t^2\frac{d^2y}{dx^2} - 2t\frac{dy}{dx} + 2y = 0 \quad \text{for } t > 0 \quad \text{is}$$

(A) $\{1\}$ (B) $\{1, -1\}$ (C) $\{1, 2\}$ (D) $\{-1, 2\}$

8. Let $g : \mathbb{R} \to \mathbb{R}$ be differentiable with $g'(x^2) = x^3$ for all $x > 0$ and $g(1) = 1$. Then $g(4)$ equals

(A) 64/5 (B) 32/5 (C) 37/5 (D) 67/5

9. Suppose X and Y are two independent random variables both following Poisson distribution with parameter λ. What is the value of $E(X - Y)^2$?

(A) λ (B) 2λ (C) λ^2 (D) $4\lambda^2$

10. If $A_1, A_2, \ldots, A_n$ are independent events with probabilities $p_1, p_2, \ldots, p_n$ respectively, then

$$P\left(\bigcup_{i=1}^{n} A_i\right)$$

equals

(A) $\sum_{i=1}^{n} p_i$ (B) $\prod_{i=1}^{n} p_i$ (C) $\prod_{i=1}^{n}(1 - p_i)$ (D) $1 - \prod_{i=1}^{n}(1 - p_i)$

11. Ravi asked his neighbor to water a delicate plant while he is away. Without water, the plant would die with probability 4/5 and with water it would die with probability 3/20. The probability that Ravi's neighbor would remember to water the plant is 9/10. If the plant actually died, what is the probability that Ravi's neighbor forgot to water the plant?

(A) $4/5$ (B) $27/43$ (C) $16/43$ (D) $2/25$

12. Suppose there are n positive real numbers such that their sum is 20 and the product is strictly greater than 1. What is the maximum possible value of n?

(A) 18 (B) 19 (C) 20 (D) 21

13. Which one of the following statements is correct regarding the elements and subsets of the set $\{1, 2, \{1, 2, 3\}\}$?

(A) $\{1, 2\} \in \{1, 2, \{1, 2, 3\}\}$ (B) $\{1, 2\} \subseteq \{1, 2, \{1, 2, 3\}\}$

(C) $\{1, 2, 3\} \subseteq \{1, 2, \{1, 2, 3\}\}$ (D) $3 \in \{1, 2, \{1, 2, 3\}\}$

14. The number of terms independent of x in the binomial expansion of $\left(3x^2 + \frac{1}{x}\right)^{10}$ is

(A) 0 (B) 1 (C) 2 (D) 5

15. The number of positive integers n for which $n^2 + 96$ is a perfect square is

(A) 0 (B) 1 (C) 2 (D) 4

16. Suppose a 6 digit number N is formed by rearranging the digits of the number 123456. If N is divisible by 5, then the set of all possible remainders when N is divided by 45 is

(A) $\{30\}$ (B) $\{15, 30\}$ (C) $\{0, 15, 30\}$ (D) $\{0, 5, 15, 30\}$

17. The number of positive integers n for which

$$n^3 + (n+1)^3 + (n+2)^3 = (n+3)^3$$

is

(A) 0 (B) 1 (C) 2 (D) 3

18. Let $A = \begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix}$, and $B = A + A^2 + A^3 + \cdots + A^{50}$. Then

(A) $B^2 = I$ (B) $B^2 = 0$ (C) $B^2 = A$ (D) $B^2 = B$

19. Let A be a real 2×2 matrix. If $5 + 3i$ is an eigenvalue of A, then $\det(A)$

(A) equals 4 (B) equals 8 (C) equals 16

(D) cannot be determined from the given information

20. Let $f : (0, \infty) \to (0, \infty)$ be a strictly decreasing function. Consider

$$h(x) = \frac{f\left(\frac{x}{1+x}\right)}{1 + f\left(\frac{x}{1+x}\right)}.$$

Which one of the following is always true?

(A) h is strictly decreasing

(B) h is strictly increasing

(C) h is strictly decreasing at first and then strictly increasing

(D) h is strictly increasing at first and then strictly decreasing

21. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$. How many functions $f : A \to A$ can be defined such that $f(1) < f(2) < f(3)$?

(A) $\binom{8}{3}$ (B) $\binom{8}{3}5^8$ (C) $\binom{8}{3}8^5$ (D) $\frac{8!}{3!}$

22. The infinite series $\sum_{n=1}^{\infty} \frac{a^n \log n}{n^2}$ converges if and only if

(A) $a \in [-1, 1)$ (B) $a \in (-1, 1]$ (C) $a \in [-1, 1]$ (D) $a \in (-\infty, \infty)$

23. Given that $\int_{-\infty}^{\infty} e^{-x^2/2} dx = \sqrt{2\pi}$, what is the value of

$$\int_{-\infty}^{\infty} |x|^{-1/2} e^{-|x|} dx?$$

(A) 0 (B) $\sqrt{\pi}$ (C) $2\sqrt{\pi}$ (D) ∞

24. Let $f : \mathbb{R} \to \mathbb{R}$ be a strictly increasing function. Then which one of the following is always true?

(A) The limits $\lim_{x \to a+} f(x)$ and $\lim_{x \to a-} f(x)$ exist for all real numbers a

(B) If f is differentiable at a then $f'(a) > 0$

(C) There cannot be any real number B such that $f(x) < B$ for all real x

(D) There cannot be any real number L such that $f(x) > L$ for all real x

25. An integer is said to be a *palindrome* if it reads the same forward or backward. For example, the integer 14541 is a 5-digit palindrome and 12345 is not a palindrome.

How many 8 digit palindromes are prime?

(A) 0 (B) 1 (C) 11 (D) 19

26. Let x and y be real numbers satisfying $9x^2 + 16y^2 = 1$. Then $(x + y)$ is maximum when

(A) $y = 9x/16$ (B) $y = -9x/16$ (C) $y = 4x/3$ (D) $y = -4x/3$

27. Consider the function

$$f(x) = \frac{e^{-|x|}}{\max\{e^x, e^{-x}\}}, \qquad x \in \mathbb{R}.$$

Then

(A) f is not continuous at some points

(B) f is continuous everywhere, but not differentiable anywhere

(C) f is continuous everywhere, but not differentiable at exactly one point

(D) f is differentiable everywhere

28. Let A be a square matrix such that $A^3 = 0$, but $A^2 \neq 0$. Then which of the following statements is not necessarily true?

(A) $A \neq A^2$

(B) Eigenvalues of A^2 are all zero

(C) $\mathrm{rank}(A) > \mathrm{rank}(A^2)$

(D) $\mathrm{rank}(A) > \mathrm{trace}(A)$

29. Suppose a is a real number for which all the roots of the equation $x^4 - 2ax^2 + x + a^2 - a = 0$ are real. Then

(A) $a < -\frac{2}{3}$ (B) $a = 0$ (C) $0 < a < \frac{3}{4}$ (D) $a \geq \frac{3}{4}$

30. A club with n members is organized into four committees so that each member belongs to exactly two committees and each pair of committees has exactly one member in common. Then

(A) $n = 4$

(B) $n = 6$

(C) $n = 8$

(D) n cannot be determined from the given information