2015

BOOKLET NO. Test Code: CSB

Afternoon Time: 2 hours

On the answer-booklet, write your Registration Number, Test Code and Number of this booklet in appropriate places.

ATTENTION!

Read the following carefully before you start.

The question paper is divided into the following two groups:

Group A: Total of 20 marks. Attempt ALL questions.

Group B: Total of 80 marks. It has five sections. Select only one section, and answer any four questions from the selected section.

The five sections are: (i) Computer Science, (ii) Electrical and Electronics Engineering, (iii) Mathematics, (iv) Physics, and (v) Statistics.

ALL ROUGH WORK MUST BE DONE ON THIS BOOKLET AND/ OR ON THE ANSWER BOOKLET. YOU ARE NOT ALLOWED TO USE CALCULATING/COMMUNICATING DEVICES OR MATHEMATICAL TABLES.

STOP! WAIT FOR THE SIGNAL TO START!

GROUP A

Answer all questions

A1 Given an array A of n positive integers, write a program segment / pseudo-code to print the histogram of A using the hash character (#). Your histogram should consist of n vertical columns of # with the i-th vertical bar containing $A[i]$ number of #'s. For example, if you consider A as the array $\{4, 1, 2, 1\}$, your output should look like

```
#
#
#     #
#  #  #  #
```

[6]

A2 Prove that in any sequence of 105 integers, there will always be a subsequence of consecutive elements in the sequence, whose sum is divisible by 99. [8]

A3 Consider all possible permutations of eight distinct elements a, b, c, d, e, f, g, h. In how many of them, will d appear before b? Note that d and b may not necessarily be consecutive. [6]

GROUP B

(i) Computer Science

C1 Consider the following C program.

```
#include <stdio.h>

main() {
  int arr[] = {1, 1, 2, 4, 8, 16, 32, 64};
  int i, j, val, t = 16;
  unsigned char c;

  for (i = 0; i < 256; i++) {
      c = i;
      val = 0;
      for (j = 0; j < 8; j++)
          val = val + ((c >> j) & 0x1)*arr[j];
          if (val == t)
             printf("%d\n", i);
  }
}
```

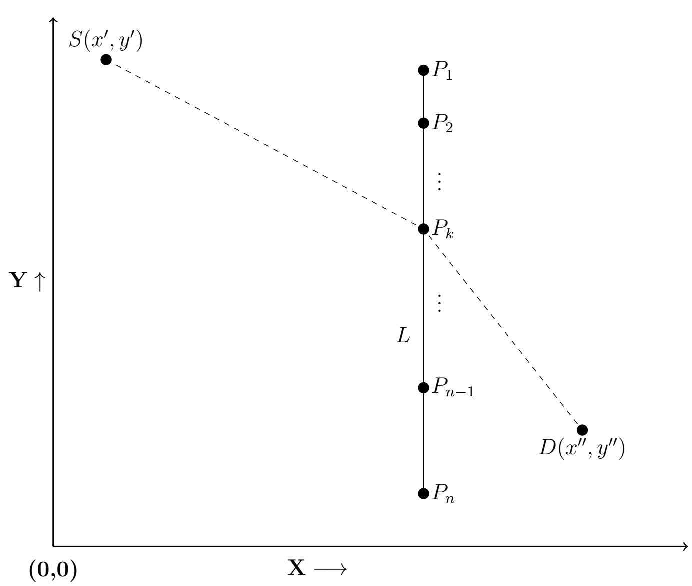
(a) Trace the execution of the code inside the `for` loop indexed by `i` when `i` = 35.

(b) What will be the output of the program? Justify your answer.

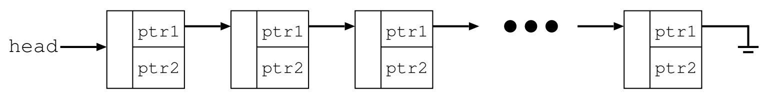
(c) What will be the output of the program if `t` = 130 (instead of 16)? Justify. [10 + 6 + 4 = 20]

C2 (a) You are given two strings S and T, each of length α, consisting only of lower case English letters (a,b,...,z). Propose an $O(\alpha)$-time algorithm to decide whether S can be obtained by permuting the symbols of T. For example, if $S = \textit{algorithm}, T = \textit{logarithm}$, your algorithm should return YES; but if $S = \textit{trainee}, T = \textit{retinaa}$, your algorithm should return NO.

(b) Let $P_1(x, y_1), P_2(x, y_2), \ldots, P_n(x, y_n)$ be a collection of n distinct points lying on a vertical line L. The value of x is stored in a variable, and $y_1, y_2, \ldots, y_n$ are stored in an array in decreasing order. Additionally, you are given two points $S(x', y')$ and $D(x'', y'')$, one on either side of L. A route R from S to D is a two-hop path $S \to P_k \to D$, where P_k is one of the points from $\{P_1, P_2, \ldots, P_n\}$. The cost of R is defined as the sum of the lengths of SP_k and P_kD. Design an $O(\log n)$-time algorithm to find the minimum-cost route from S to D, i.e., your task is to select an appropriate point P_k on L such that the cost of the route R from S to D through P_k is minimized. [8+12=20]



C3 (a) Consider a linked list containing n nodes, where each node contains two pointers `ptr1` and `ptr2`. For each node, `ptr1` points to the next node of the list. Describe how pointer **ptr2** should be set up for each node so that you will be able to locate the i-th node from the start node in the list traversing no more than $\lceil \log i \rceil + \lceil i/2 \rceil$ nodes.



(b) You are given a logic block L that takes two inputs A and B, and produces $\overline{A}+B$ as output. Realize a two-input XOR gate using only the logic block L. You can use as many pieces of block L as you need. You may use the constant function 0; but no other type of gate is allowed.

[12+8=20]

C4 (a) Consider the use of Cyclic Redundancy Code (CRC) with generator polynomial $G(x)$ for error detection. Recall that error detection with a *CRC* works by appending the *CRC* value to the bit sequence to make it a multiple of $G(x)$.

(i) Calculate the *CRC* value of the bit sequence 1 1 0 0 1 1, if $G(x) = x^4 + x^3 + 1$.

(ii) A *burst error* of length k means that there are k bits from the first to the last error positions in the frame, including both positions. Note that the intermediate bits may or may not be in error. For example, if 1 0 1 1 0 0 is transmitted and 1 1 0 1 1 0 is received, then we can say that a burst error of length 4 has occurred.

Construct a burst error of length 5 in such a way that the error cannot be detected by the *CRC* with the $G(x)$ given above.

(b) Stations A and B are connected through a line of bandwidth 64 *kbps*. Station A uses 16 byte packets to transmit messages to B using a sliding window protocol. The round trip propagation delay between A and B is 50 milliseconds. Determine the window size A should use to maximize the line utilization. Assume that the `ack` frame is of negligible size and processing delay may be ignored. Justify your answer. [(5+7)+8=20]

C5 (a) Construct two nonregular languages L_1 and L_2 such that $L_1 \bigcup L_2$ is regular.

(b) Prove that the languages L_1 and L_2 constructed above are nonregular and $L_1 \bigcup L_2$ is regular.

(c) Construct a context free grammar (CFG) to generate the following language:
$L = \{a^n b^m c^{n+m} : n, m \text{ are integers, and } n \geq 1, m \geq 1\}$.

[4 + (3+3+2) + 8 = 20]

C6 (a) Consider the following timings for a five-stage processor pipeline (these timings include the latching overhead):

Fetch	305 ps
Decode	275 ps
Execute	280 ps
Memory	305 ps
Write Back	250 ps

(i) Given the timings for the datapath stages listed above, what would be the clock period for the entire datapath?

(ii) In a pipelined datapath, assuming no hazards or stalls, what will be the throughput (instructions per second) in steady state?

(iii) Assuming that N instructions are executed, and that all N instructions are `add` instructions, what is the speedup of this pipelined implementation compared to a non-pipelined implementation? Assume that each `add` instruction consists of Fetch, Decode, Execute and Write

Back.

(b) Consider scheduling n processes $P_1, P_2, \ldots, P_n$ which are created in this order at almost the same instant. Assume that all processes have exactly one CPU burst of duration D units (and no I/O bursts). Compute the average waiting time and average turn-around time if the scheduling policy is:
(i) FCFS, (ii) RR with time slice d units ($d < D$).
Assume that it takes δ units of time to switch from one running process to another and Δ units of time to switch from a terminated process to a running process. [(3+3+6)+(4+4)=20]

C7 Consider the following schema:

SUPPLIER (<u>supId</u> : integer, supName : string, supAddress : string)

PARTS (<u>partId</u> : integer, partName : string, partColour : string)

CATALOG (<u>supId</u> : integer, <u>partId</u> : integer, price : real)

The key fields are underlined, and the domain of each field is listed after the field name. The CATALOG relation lists the prices charged for parts by suppliers.

(a) Let the relations have the following properties:

Relation	**Total no. of tuples**	**No. of tuples per block**
SUPPLIER	2,000	25
PARTS	4,500	30
CATALOG	9,000	45

Estimate the number of block accesses required to produce the result of the following query:
Find the names of suppliers who supply every part.

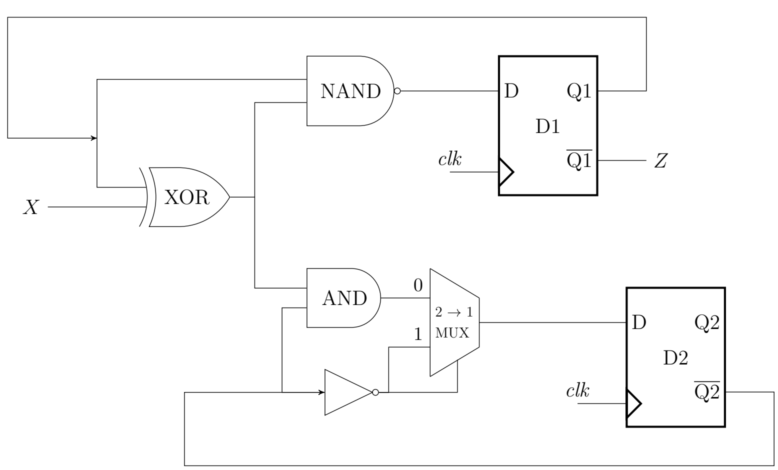
(b) Write the above query (given in (a)) in relational algebra using some or all of the following operators: SELECT, PROJECT, JOIN, CARTESIAN PRODUCT, UNION, INTERSECTION, DIFFERENCE. [10+10=20]

(ii) Electrical and Electronics Engineering

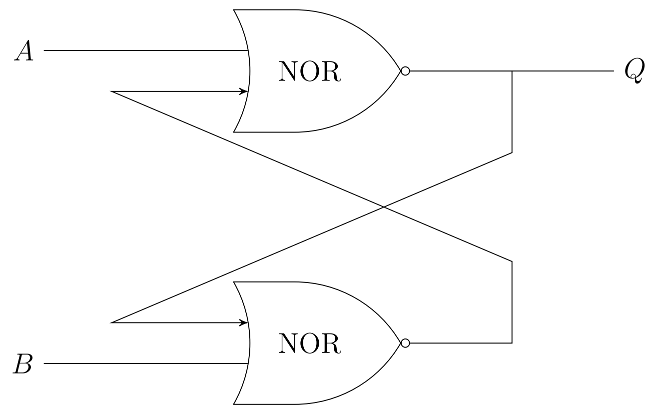
E1 (a) You are required to design a 4-bit prime number checker. Note that 0 and 1 are not prime. Design the circuit using a single $4 \rightarrow 1$ multiplexer and a minimal number of AND, OR or NOT gates, if needed.

(b) Design a combinational logic circuit that takes an unsigned 2-bit integer as input and computes its square. [10+10=20]

E2 (a) Consider the synchronous sequential circuit (shown below) with two edge-triggered D flip-flops, which are clocked with the clock signal clk. The lines X and Z denote the primary input and output, respectively. Construct the state diagram of the circuit.

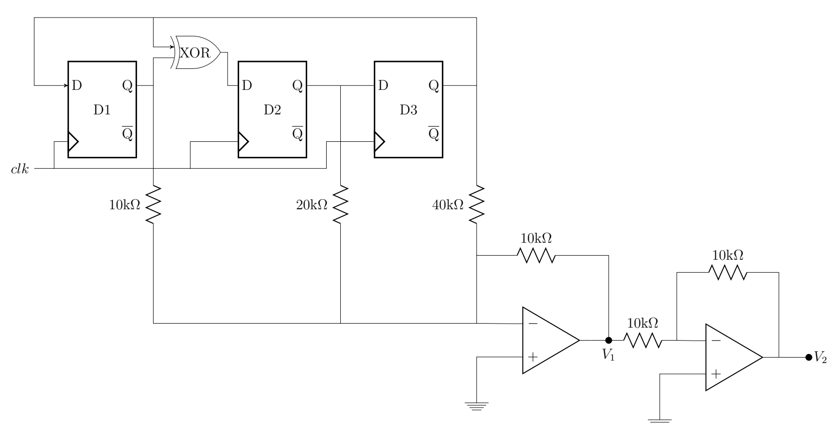


(b) Two NOR gates are cross-connected to form a latch as shown in the following figure. We set $A = B = 1$, and the circuit is allowed to become stable. Next, we set $A = B = 0$. Choose one of the following as the output at Q: (i) 0, (ii) 1, (iii) stable, but cannot be predicted, (iv) depends on previous inputs, (v) oscillates between 0 and 1, (vi) none of these. Justify your answer.



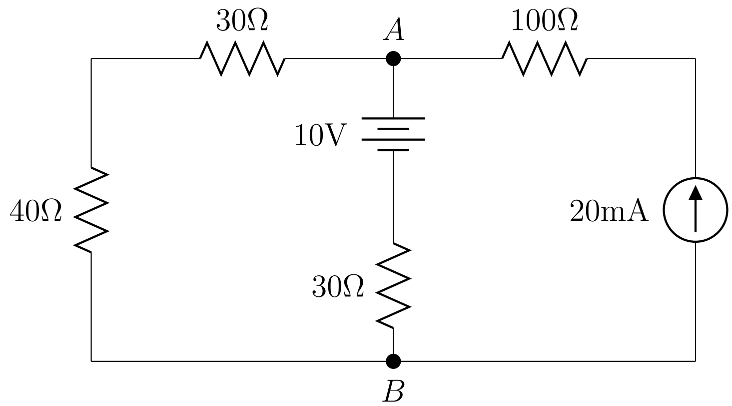
[14+6=20]

E3 Consider the following circuit with two ideal OP-AMPs. In this circuit, D_1, D_2, and D_3 represent three edge-triggered D-flip-flops, which are synchronized with the same clock signal clk. The flip-flops are initialized as: $D_1, D_2, D_3 = 0, 1, 0$. Assume that logic 1 represents 4V and logic 0 represents 0V. Compute the output voltage V_1, V_2 (in volts) and show the voltage waveforms for five consecutive clock pulses applied to the clk line.

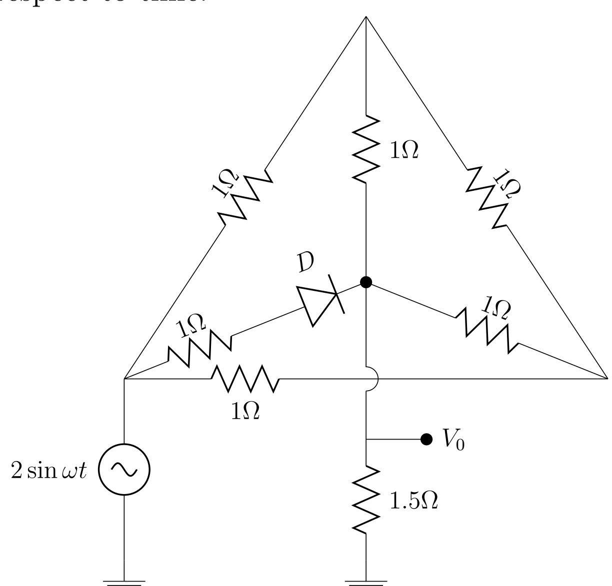


[14+6=20]

E4 (a) In the following circuit, determine the voltage difference V_{AB} between the points A and B.



(b) Consider the following circuit where D represents an ideal diode. Compute the voltage V_0 and draw the waveform with respect to time.



[8+12=20]

E5 (a) Consider a binary symmetric channel with transition probability 0.4. The channel has two input symbols ($x_0 = 0, x_1 = 1$) and two output symbols ($y_0 = 0, y_1 = 1$). Let the probability of choosing x_0 be 0.5. Calculate (i) the average mutual information between the channel input and channel output, and (ii) the channel capacity.

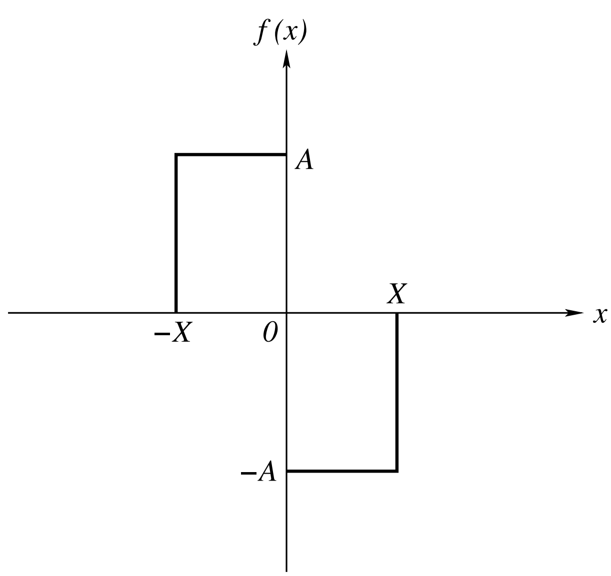
(b) Consider a discrete memoryless source with source alphabet $\Sigma = \{\alpha, \beta, \gamma\}$ with probabilities

$$p_\alpha = 0.25; \quad p_\beta = 0.50; \quad p_\gamma = 0.25.$$

Devise an optimum variable-length code for the second-order extension of the source with strings of 0's and 1's based on Huffman coding. Calculate the corresponding entropy and average code-word length.

[(4+2)+(8+3+3)=20]

E6 (a) Consider the following function $f(x)$.



Compute its Fourier transform and plot the Fourier spectrum.

(b) Find the inverse Laplace transform of $F(s)$ defined as

$$F(s) = \frac{e^{-5s}(5s-2)}{s^2-4s+13}.$$

(c) Given two discrete time sequences

$$f(t) = 3\delta(t) + 2\delta(t-1) - 5\delta(t-2)$$

$$g(t) = \delta(t) - 4\delta(t-1) + 2\delta(t-2)$$

calculate the sequence $h(t)$, where

$$h(t) = f(t) * g(t).$$

Note that $*$ denotes the convolution operation and $\delta(t)$ the Dirac delta function. [10+5+5=20]

E7 (a) For a 3-phase induction motor, the line current does not exceed three times the full-load current. The short circuit current is 6 times the full-load current and the full-load slip is 5%. Find (i) a suitable auto-transformation ratio for starting the motor, and (ii) the starting torque in terms of full-load torque.

(b) A 2200kVA, 440V, 50Hz transformer has power factor 0.81. The transformer works with maximum efficiency of 99% at half-load. Suppose the transformer is on full-load for 8 hours, on half-load for 6 hours and on one-tenth of load for the rest of the day. Calculate the all-day efficiency of the transformer. [10+10=20]

(iii) Mathematics

M1 (a) If $n \geq 1$ is an integer, show that among $n, n+1, n+2$ and $n+3$, there is one which is co-prime to the other three.

(b) Let m and n be two distinct positive integers. If p is an odd prime and p divides $m^p + n^p$, then show that p divides $m+n$. Hence, or otherwise, show that p^2 divides $m^p + n^p$.

[8+(6+6)=20]

M2 Let $\mathbb{Q}$ denote the additive group of rational numbers. For integers $n \geq 1$, let P_n denote the subgroup of $\mathbb{Q}$ generated by $\frac{1}{n!}$.

(a) Show that $P_1 \subseteq P_2 \subseteq \cdots$. Are the inclusions proper? Justify your answer.

(b) Show that $\mathbb{Q} = \bigcup_{n \geq 1} P_n$. Hence, or otherwise, show that $\mathbb{Q}$ is not cyclic. [(6+4)+(6+4)=20]

M3 Let $\{f_n\}$ be a sequence of non-decreasing functions from $[0,1]$ to $[0,1]$ such that

$$\lim_{n \to \infty} f_n(x) = f(x)$$

pointwise for every $x \in [0,1]$ for some continuous function f.

(a) Show that $\{f_n\}$ converges uniformly.

(b) If the non-decreasing condition on $\{f_n\}$ is dropped, does the assertion in (a) still hold? Justify your answer by proving it or by giving a counter-example. [14+6=20]

M4 (a) Find the number of simple directed graphs that can be constructed with vertex set $\{v_1, v_2, \ldots, v_n\}$ such that if, in a graph, there exists an edge $v_i \to v_j$, then there is no edge $v_j \to v_i$.

(b) A transitive tournament is an orientation of a complete graph G with vertex set $\{v_1, v_2, \ldots, v_n\}$ in which there exists a directed edge $v_i \to v_j$ iff $i < j$. Show that every orientation of the complete graph G contains a transitive tournament on $[\log_2 n]$ vertices, where $[x]$ is the largest integer less than or equal to x. [7+13=20]

M5 For any square matrix C with real entries, define $\lambda_1(C)$ as the maximum of the absolute values of the eigenvalues of C. Two square matrices $A_{n\times n}$ and $B_{n\times n}$ with real entries are said to satisfy property X if

$$\lambda_1(AB) \leq \lambda_1(A)\lambda_1(B).$$

(a) For symmetric matrices A and B with $AB = BA$ show that the property X holds.

(b) If the condition of symmetry is dropped, does the property X still hold? Justify your answer.

(c) Give an example of two square matrices A and B for which the property X does not hold. [7+7+6=20]

M6 Consider $f_c(x) = \frac{1}{4} + \frac{1}{c}(x - x^2)$ for some real $c > 0$. Define the sequence $S = \{x_n\}_{n\geq 0}$, starting with an arbitrary x_0 and $x_{n+1} = f_c(x_n)$ for all $n \geq 0$. Let $A = \{x_0 : S \text{ converges}\}$ and for all $x \in A$, let $\ell(x)$ be the corresponding limit.

(a) If $x_0 = 0$, show that the sequence corresponding to $c = 1$ is increasing and bounded above. Find its limit $\ell(0)$.

(b) Find the set $\{\ell(x) : x \in A\}$ corresponding to $c = 1$.

(c) Find all values of c such that the sequence $\{x_n\}$ converges for at least one starting value x_0. [7+7+6=20]

M7 (a) An ideal I in a ring R is said to satisfy property A if the following holds:

$$\text{For } a, b \in R, \text{ if } a \cdot b \in I, \text{ then either } a \in I \text{ or } b \in I.$$

(i) Let I_1, I_2 be two distinct ideals in a ring R such that both satisfy property A and none is contained in the other. Prove that $I_1 \bigcap I_2$ does not satisfy property A.

(ii) Let I be an ideal in a ring R satisfying property A. Moreover, assume that I contains no (non-zero) zero divisor. Prove that R is an integral domain.

(b) Consider the ring $C[0, 1]$ of real valued continuous functions on $[0, 1]$ under pointwise addition and multiplication. Let $0 < a < 1$ and $M_a = \{f \in C[0, 1] \ : \ f(a) = 0\}$. Prove that M_a is not equal to the principal ideal generated by $x - a$.

[(7+7)+6=20]

(iv) Physics

P1 A particle of mass m is thrown upward with a speed so that it reaches a maximum height of 400 feet and then returns to its starting point. Obtain Hamilton's equations of motion for the upward and downward travels separately. Using these, find the initial speed and the time taken by the particle to return to the starting point. [Acceleration due to gravity is $32 feet/sec^2$]

[(4+4)+(6+6) = 20]

P2 (a) An infinitely long wire having a circular cross-section of radius a carries current I. Consider a point P having cylindrical co-ordinates (r, ϕ, z) at a perpendicular distance r from the axis of the wire. The magnetic permeability inside the medium of the wire is μ_1, whereas outside the wire it is μ. Find the vector potential $\overrightarrow{A}$ (having the z-component only) and hence the corresponding magnetic flux density $\overrightarrow{B}$ (having the ϕ-component only) at the chosen point P when,

(i) P is inside the wire,

(ii) P is outside the wire.

Use the boundary conditions :
$A_z = 0$ at $r = a$; B is continuous across $r = a$.

(b) A parallel plate capacitor with distance of 1 cm between the plates and the plate area of $0.2m^2$ has a potential of $1000V$. Calculate the work done in slowly pulling the plates apart to double this spacing. (Permittivity of free space $\epsilon_0 = 8.9 \times 10^{-12} C^2/Nm^2$)

[14+6=20]

P3 (a) A hydrogen like atom X has a stationery nucleus of charge Ze where e is the charge of an electron, Z is a numerical constant. $6.75E_0$ eV energy is needed to excite an electron from the second Bohr orbit to the fourth one, where E_0 is the ionization energy of the hydrogen atom in the Bohr model. Determine:

(i) The value of Z.

(ii) The radius of the first Bohr orbit of the atom X.

(iii) The wavelength of the electromagnetic radiation (photon) to completely remove an electron from the first Bohr orbit of X.

(b) Calculate the potential difference through which an electron must be accelerated in a X-ray tube in order that the short wave limit of the continuous X-ray spectrum produced shall be exactly 1 Å.

Given $E_0 = 13.60eV$; Planck's constant $h = 6.62 \times 10^{-34}$ Joule second; velocity of light $c = 3 \times 10^8$ meters/second; Bohr radius of the hydrogen atom $= 5.3 \times 10^{-11}$ meter, $1eV = 1.6 \times 10^{-19}$ Joules; Charge of an electron $e = 1.60 \times 10^{-19}$ Coulumb. [(8 + 4 + 4) + 4 = 20]

P4 (a) Consider the following wave function of a particle of mass m moving freely inside a 1-dimensional infinite potential well of length a at time $t = 0$:

$$\psi(x) = \frac{2}{\sqrt{a}} \cos(\frac{\pi x}{a}) \sin(\frac{2\pi x}{a})$$

Find the wave function and energy expectation value at time $t = T$.

(b) Consider the following self-adjoint operator A:

$$A = \sqrt{\frac{1}{3}}\sigma_z + \sqrt{\frac{2}{3}}\sigma_x$$

where σ_z and σ_x are usual Pauli matrices.

(i) Show that $A^2 = I$, where I is the identity matrix.

(ii) Find the eigenvalues of A.

(c) Consider a simple harmonic oscillator. Show that the expectation value of kinetic energy and potential energy are the same for any energy eigenstate of this oscillator.

[(4+2)+(3+3)+8=20]

P5 (a) A material is brought from temperature T_i to T_f by placing it successively in contact with a series of N reservoirs at temperature $T_i + \delta T, T_i + 2\delta T, \cdots, T_i + N\delta T = T_f$. Assume the heat capacity C of the material is temperature independent. Calculate the entropy change of the total system (material plus reservoirs). What is the entropy change for $N \to \infty$ for fixed $T_f - T_i$?

(b) For a diatomic ideal gas at room temperature, what fraction of the heat supplied is available for external work if the gas is expanded at (i) constant pressure and (ii) constant temperature?

(c) Prove that $\left(\frac{\partial P}{\partial S}\right)_T \left(\frac{\partial S}{\partial T}\right)_P = -\left(\frac{\partial P}{\partial T}\right)_S$
where the thermodynamic variables P, V, T and S carry their usual meanings. [8 + (4+4) + 4 = 20]

P6 (a) A system consists of N weakly interacting particles at a temperature sufficiently high such that the classical statistics are applicable. Each particle has mass m and oscillates in one direction about its equilibrium position. Using the idea of Virial Theorem calculate the heat capacity at temperature T in the following two cases:

(i) The restoring force is proportional to the displacement x from the equilibrium position.

(ii) The restoring force is proportional to x^3 where x is the displacement from the equilibrium position.

(b) Argon gas (molecular weight 40) is contained in a chamber at a temperature T=300K. Using the Maxwell velocity distribution function for the gas molecules, calculate the most probable velocity v_p.

(c) The adiabatic expansion of an ideal gas is described by the equation $PV^\gamma = C$, where γ and C are constants. Calculate the work done by the gas in expanding adiabatically from the state (V_i, P_i) to (V_f, P_f), where the symbols have their usual meaning. [(6+6)+4+4=20]

P7 (a) Draw the current voltage characteristics of a Zener diode, mark the knee-voltage (V_{z_o}), and write down the expression for Zener voltage (V_z) at an operating current I_z where r_z, the reciprocal of the slope of the straight line in the diagram, is termed as dynamic resistance.

(b) Draw the equivalent circuit of an (i) ideal and (ii) non-ideal Zener diode.

(c) In the given circuit diagram the Zener diode is non-ideal. It has a knee-voltage $V_{z_o} = 9V$ and the dynamic resistance $r_z = 5\Omega$. If the supply voltage V_s varies from $15V$ to $30V$, determine the range of the output voltage.

(d) Interpret the result obtained in (c), to explain why Zener diodes are widely used as voltage reference diodes.

[(2+1+2)+(2 + 2)+8+3=20]

(v) Statistics

S1 (a) Let X, Y be iid $U(0,1)$. A fair coin is tossed. If head appears then we define

$$W = \min\{X, Y\} \text{ and } V = \max\{X, Y\},$$

otherwise, we define

$$W = \max\{X, Y\} \text{ and } V = \min\{X, Y\}.$$

Find the joint density function of (W, V).

(b) $X_1, X_2, ..., X_n$ are iid $U(\theta-1, \theta+1)$, where θ is a real number. Show that

$$\alpha(X_{(n)} - 1) + (1 - \alpha)(X_{(1)} + 1)$$

is a maximum likelihood estimator of θ for any $\alpha \in (0, 1)$. Here $X_{(1)}$ and $X_{(n)}$ denote, respectively, the minimum and maximum of X_i's. [10+10=20]

S2 Let D_1 and D_2 be two disks of radii 6 with centres at (10,0) and (20,0), respectively. Let $p_i(x, y)$ denote the uniform probability density over D_i for $i = 1, 2$. Consider a binary classification problem where $p_1(x, y)$ and $p_2(x, y)$ are the probability density functions of the two classes. Let the prior probabilities of the classes be equal. (i) Find the optimal Bayes' rule for this classification problem. (ii) Find the Bayes' risk for this rule. (iii) Is the Bayes' decision boundedly unique? Justify your answer. [5+5+10=20]

S3 A rectangle has dimensions a and b, where $a, b > 0$ are unknown. The dimensions are measured independently to produce the measurements X and Y where $X \sim N(a, 1)$ and $Y \sim N(b, 1)$. Based on these measurements we want to test the hypothesis H_0 : perimeter = 10 against H_1 : perimeter $\neq$ 10.

(a) Which of H_0, H_1 is/are simple hypothesis? Why?

(b) Find the likelihood ratio test for testing H_0 against H_1 at 5% level of significance.

(c) Suppose that after making the measurements, we get the additional information that the rectangle is a square. How will you update your test to incorporate this extra information?

[2+10+8=20]

S4 We have four unknown weights a, b, c, d and an imperfect weighing balance. By four independent weighings we have learned that

$$\begin{aligned} a &\approx c+2, \\ a+b &\approx c+d+5.1, \\ a &\approx b+1, \\ b &\approx d+2. \end{aligned}$$

It is known that the error = LHS-RHS involved in each measurement follows $N(0, \sigma^2)$ distribution with some common unknown $\sigma^2 > 0$.

(a) Find the best linear unbiased estimators (BLUE) of a, b, c, d based on these measurements. If BLUE does not exist for some of these, then explain why.

(b) Find the maximum likelihood estimator (MLE) of σ^2 based on these measurements. If MLE does not exist, then explain why. [16+4=20]

S5 (a) Give an example of a bivariate probability distribution function $F(x, y)$ satisfying the following two conditions:

(i) $F(x, y)$ is not continuous, and

(ii) if (X, Y) is jointly distributed random variables with this distribution function, then for all $a, b \in \mathbb{R}$ we have $P(X = a, Y = b) = 0$.

(b) There are 10 urns. The i-th urn contains i red balls and $i+2$ white balls for $i = 1, ..., 10$. Two distinct urns are chosen at random. A ball is selected at random from the first urn and transferred to the second, and then a ball of the opposite color is transferred from the second to the first urn. Now an urn is again chosen at random independently. A ball is chosen at random from it. What is the chance that this ball is red? [10+10=20]

S6 Consider the following trivariate data set consisting of 13 points:

- (0,0,0),
- $(\pm 1, 0, 0)$,
- $(0, \pm 1, 0)$, $(0, \pm 2, 0)$,
- $(0, 0, \pm 1)$, $(0, 0, \pm 2)$, $(0, 0, \pm 3)$.

Find the variance-covariance matrix of this data set. Compute the principal components based on this variance-covariance matrix. If two new points of the form $(-\delta, 0, 0)$ and $(\delta, 0, 0)$ are added to the data set for some $\delta > 1$, then find the smallest δ_0 such that the first principal component for $\delta < \delta_0$ is different from that for $\delta > \delta_0$. [5+7+8=20]

S7 (a) A random variable X can take the values -1, 0 and 1 with unknown probabilities p, q and r, respectively, where $p+q+r = 1$ and $p, q, r > 0$. We have drawn a sample X_1 of size 1 from this distribution. Consider the statistics $S(X_1) = X_1$ and $T(X_1) = |X_1|$. Is $S(X_1)$ a complete statistic for the parameter (p, q, r)? Is $T(X_1)$ a complete statistic for the parameter (p, q, r)? Justify your answers.

(b) In this setup, find (if possible) a complete statistic that is not sufficient. Also, find (if possible) a sufficient statistic that is not complete. Justify your answer. [(5+5)+(5+5)=20]