2016

Booklet No. TEST CODE: QEB

Afternoon

Questions: 10	**Time: 2 hours**

- *On the answer booklet write your Name, Registration number, Test Code, Number of the booklet etc. in the appropriate places.*
- This test has 10 questions. Answer any 4. All questions carry equal marks (25).

1. Consider a market consisting of 600 buyers and 400 sellers of used cars. There are 100 bad quality used cars (lemons) and 300 good quality used cars (peaches). Suppose the valuation of a lemon is 20 for both a buyer and a seller. The valuation of a peach, however, is 100 for a buyer and 60 for a seller.

   (a) What is the efficient outcome?

   (b) Solve for the market outcome when the identity of all cars, i.e. whether a car is lemon or a peach, is common knowledge. Illustrate the outcome graphically.

   (c) Next, suppose that the identity of any car is private knowledge of its owner. What is the market outcome? Illustrate the outcome graphically.

   (d) Suppose any lemon owner, at a cost of 10, can transform a lemon into a peach. Will she have an incentive to do that? Explain.

   [5+5+10+5=25]

2. Consider a farmer who can pump groundwater from an aquifer beneath his land to which no one else has access. The profit from water pumped to the surface is

$$f(X) - cX$$

   where X is the number of litres of diesel used for pumping water, c is the cost of a litre of diesel used for pumping, and the revenue function f is differentiable, strictly increasing, and strictly concave with $f'(0) > c$ and $f'(k) < c$ for some $k > 0$.

   (a) Characterize the profit-maximizing value of X.

   (b) Now suppose the number of farmers in the area with access to the aquifer who could also pump water from it is effectively unlimited. The profit for farmer i is now given by

$$\pi_i = \frac{x_i}{X} f(X) - cx_i,$$

where x_i is the diesel used by farmer i and X is the total litres of diesel used by all farmers who pump water. Characterize the total profit and the total litres of diesel that will be used, assuming that each farmer tries to maximize his own profit. Are the total profit and the total diesel used larger or smaller or the same as in the case of a single farmer in the previous part? Why?

(c) Now suppose the number of farmers who can pump water from the aquifer is a fixed finite number N. Characterize the litres of diesel that will be used by each of the N farmers. Is the total profit of all farmers larger or smaller or the same as the corresponding values in the previous two cases? Why?

[5+10+10=25]

3. Suppose agents need to borrow to invest in a project.There are two types of borrowers, risky and safe, characterised by the probability of success of their projects, p_r and p_s respectively, where $0 < p_r < p_s < 1$. Risky and safe borrowers exist in proportions θ and $1 - \theta$ in the population. Suppose a risky investment project requires one unit of capital.The outcome of the project is either a success (S) or failure (F).

   The return of a project of a borrower of type i is $R_i > 0$ if successful and 0 if it fails. Assume that risky and safe projects have the same mean return, that is, $p_r R_r = p_s R_s \equiv \bar{R}$, but risky projects have a greater spread around the mean. Borrowers are risk-neutral and maximise expected returns. Borrowers of both types have an reservation payoff $\bar{u}$.

   The lending side is represented by risk-neutral banks whose opportunity cost of capital is $\rho \geq 1$ per unit. There is a standard debt contract between a borrower and the bank with a fixed repayment (principal plus interest) in case of success and zero repayment in case of failure.

Assumptions:

(A1)

$$\bar{R} > \rho + \bar{u}.$$

(A2) The credit market is competitive so that banks are subjected to a *zero-profit constraint* on each loan.

(a) First, to set the benchmark, consider that the bank has *full information* about a borrower's type.

(i) Write down the expressions for expected profits of the bank from each type of borrower.

(ii) Write down the expressions for expected payoffs to each type of borrower.

(iii) Given the zero-profit constraint on each loan, what repayment (principal plus interest), r_i, $i = r, s$, will the bank offer to type i borrower? Will the type i borrower accept this offer? Give clear explanations for your answers.

(iv) What will be the average repayment rate (rate at which the bank gets repaid)?

(b) Now consider that the bank cannot identify a borrower's type. Explain clearly that the *separating* repayments that you have identified in part (a) will *not* work.
[4+4+4+4+9=25]

4. Consider a Solow-Swan type economy with the aggregate CES production function given by

$$Y = F(K, L) = A\left[a(bK)^{\psi} + (1-a)[(1-b)L]^{\psi}\right]^{\frac{1}{\psi}}$$

where Y denotes output, K denotes the capital stock, L denotes labor supply, and $0 < a < 1$, $0 < b < 1$ and $\psi < 1$. Let $y = \frac{Y}{L}$ and $k = \frac{K}{L}$. Assume that the population grows at a constant rate and capital depreciates at a constant rate.

(a) Show that the intensive form production function is given by

$$y = f(k) = A\left[a(bk)^{\psi} + (1-a)(1-b)^{\psi}\right]^{\frac{1}{\psi}}$$

(b) Derive an expression for

1. $f'(k)$
2. $\frac{f(k)}{k}$

Are $f'(k)$ and $\frac{f(k)}{k}$ positive and diminishing for all values of ψ?

(c) Draw a picture that describes the dynamic behavior of the above CES economy (on the y-axis, put $\frac{\dot{k}}{k}$, and on the x-axis put k). Assume $0 < \psi < 1$. Can this economy generate endogenous steady state growth? Why or why not?

(d) Re-do the same exercise in part (c), although now assume that $\psi < 0$? What happens to the economy over time?

(e) Does the model exhibit absolute convergence? Explain your answer by first defining what absolute convergence is.

[3+7+6+6+3=25]

5. Suppose the production function takes the form

$$Y_t = (B_t K_t)^{\alpha}(A_t L_t)^{1-\alpha}$$

where Y denotes aggregate output, K denotes the capital stock, L denotes labor supply, $A > 0$ and $B > 0$ are constants and $\alpha \in (0, 1)$. Both capital and labor augmenting technological progress rates are given by $\frac{\dot{B}}{B} = g_B$, and $\frac{\dot{A}}{A} = g_A$ respectively. Assume that the population grows at a constant rate and capital depreciates at a constant rate.

(a) Derive the law of motion for the capital-labor ratio, $\frac{\dot{Z}}{Z}$, in effective units, where $Z = \frac{BK}{AL}$

(b) Show that the economy has a balanced growth path, i.e., a constant Z solution, if and only if $g_B = 0$, or in other words, technological progress is labor augmenting.

[10+15=25]

6.(i) An extensive form game with two players is shown in Figure 1. The actions available to each player at a decision node are shown on edges from those decision nodes. The payoffs of the players are shown in the terminal nodes, where the first component of the payoff vector belongs to Player 1 and the second component belongs to Player 2. Answer the following questions.

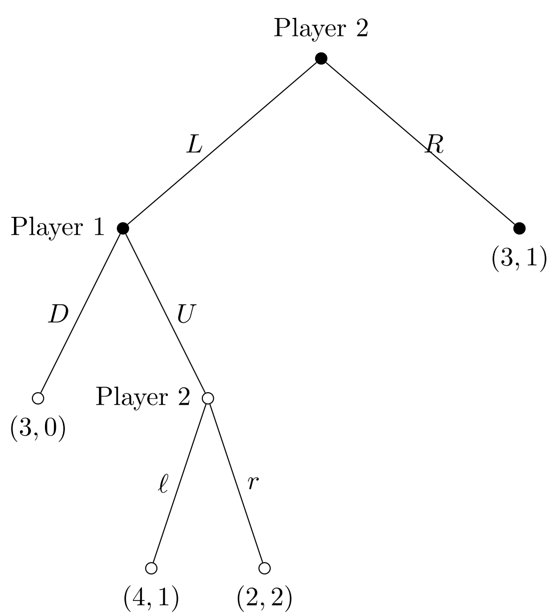

Figure 1: An extensive form game

(a) Write down the pure strategies of both the players?

(b) Describe a subgame perfect equilibrium of this game?

6.(ii) Suppose a father wants to allocate his land among his two sons $\{1, 2\}$. Utilities for the land for son 1 and 2 are u_1 and u_2 respectively - both u_1 and u_2 are non-negative real numbers. The father asks each son to announce a non-negative real number simultaneously. The son who announces the higher number gets the land (ties are broken in favor of son 1). The son who gets the land neither receives any payment nor pays anything. The son who does not receive the land is paid an amount equal to the announcement of the other son. Assume that utilities over transfers are quasilinear: if son $i \in \{1, 2\}$ gets the house he gets a net utility of u_i and son $j \neq i$ gets a net utility of p_i, where p_i is the announcement of son i.

Show that announcing u_k is a weakly dominant strategy for son $k \in \{1, 2\}$.
[5+5+15=25]

7.(i) Let $\langle N, S_1, \ldots, S_n, \pi_1, \ldots, \pi_n \rangle$ be a normal form game where $N = \{1, \ldots, n\}$ is the set of players, S_i and $\pi_i : S_1 \times \ldots \times S_n \to \Re$, $i \in N$ are the strategy set and payoff function respectively for player i. When is a strategy $s_i^* \in S_i$ a (weakly) dominant strategy for player i?

7.(ii) Three families reside on three floors of an apartment building. They are deciding whether to employ a security guard at the entrance to the building. Assume that the guard costs 1 per family. Each family has a value (not including the cost of hiring) of the guard of either 2 or -2 (it can be negative because some families resent the loss of privacy). If the guard is not hired, all families receive a net utility of zero.

The families adopt the following procedure: they report their value (i.e.. either 2 or -2) and the guard is hired if the number of families reporting the positive value, is at least α where α is either one, two or three. Note that different values of α represent different procedures - if α is one, the guard is hired as if at least one family wants it, if α is two, a majority is required and α is three would mean that hiring takes places only if the families are unanimous. Show that reporting their value truthfully is a weakly dominant strategy for each family, for all values of α.

7.(iii) Consider the problem of choosing α. Assume that each family's value is drawn independently from the uniform distribution (i.e 2 and -2 each with probability $\frac{1}{2}$). Fix a value of α. What is the expected *sum* of payoffs of all families when each family reports truthfully? What value of α maximises this expected sum?
[5+10+10=25]

8. Let the market demand function of a commodity be given by $Q = \alpha_0 + \alpha_1 P + \alpha_2 I$ where I denotes income, P denotes price and Q quantity. Let the marginal cost of the industry supplying Q, denoted by mc, be $mc = \beta_0 + \beta_1 Q + \beta_2 w$ where w is the cost of the only input - labour.

(a) Assume that the industry is perfectly competitive. You are given a data set containing data from n markets selling this commodity. The data set contains information on P, Q, I and w prevailing in each market (but no marginal cost data). Write down an econometric strategy to consistently estimate the coefficients of the demand and marginal cost functions. Explicitly list any assumptions you are making.

(b) Now suppose you do not know whether the industry is perfectly competitive or a monopoly. Let $P + \lambda \frac{\partial P}{\partial Q} Q = mc$ where $\lambda = 0$ characterizes perfect competition and $\lambda = 1$, a monopoly.

You want to estimate λ. Given the same data as part (a), can you estimate λ? Explain your intuition graphically.

(c) Will the answer to (b) change if the marginal cost is invariant to Q? Explain.

[5+15+5=25]

9. Suppose a government is planning to introduce an electrification program to supply electricity to non-electrified rural areas in a phased manner. In Phase 1, the districts with lowest literacy levels will be electrified followed by the remaining districts in Phase 2. You want to evaluate the impact of the electrification program on households' monthly per capita income as a measure of poverty. You have data available only for the cross section of households in Phase 1.

   Let $E_i = 1$ if district i is electrified in Phase 1 and 0 otherwise PC_i is the average household percapita income in district i.

   (a) Write down a simple OLS model for estimating this impact of the program. Clearly explain all the variables in your model.

   (b) Under what condition(s) is the coefficient on E_i inconsistent. Show by deriving its expected value.

   Now suppose you are able to obtain data on household's income and other characteristics for both Phase 1 and 2. Let $t = 1$ if your observations are for Phase 2 and 0 otherwise.

   (c) Using the panel data available to you now, write down an econometric specification to estimate the electrification program's impact on PC_i.

   (d) Under what condition(s) will the estimated program impact in (c) be consistent? Explain.

   [2+10+10+3=25]

10.(i) State clearly whether the following statements are TRUE or FALSE or UNCERTAIN. Give explanations in support of your answers.

(a) The autocorrelation function of an AR(1) process can never be exponential in nature.

(b) If a time series follows a white noise process, it may as well be non-stationary in nature.

(c) An MA(2) process always has zero autocorrelation for lags $\geq$ 3.

(d) A weakly (covariance) stationary time series may be strongly stationary.

10.(ii) Let a time series $\{X_t\}$ be defined by

$$X_t + \alpha_1 X_{t-1} + \alpha_2 X_{t-2} = e_t,$$

where $\{e_t\}$ is a white noise process with mean 0 and variance σ^2. Further, suppose that the roots r_1 and r_2 of a quadratic equation $Z^2 + \alpha_1 Z + \alpha_2 = 0$ are real and less than one in absolute value. Then show that a new time series $\{Y_t\}$ defined as $Y_t = X_t - r_1 X_{t-1}$ follows a first order autoregressive process with parameter r_2.
[3+3+3+3+13=25]