2015

BOOKLET No. **TEST CODE: PQB**

Afternoon

Time: 2 hours

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On the answer-booklet write your Name, Registration Number, Test Code, Number of this Booklet, etc. in the appropriate places.

There are two parts in this booklet as detailed above. Candidates having Statistics/Mathematics background are required to answer questions from Part I only as per instructions given. Those having Engineering background are required to answer questions from Part II only as per instructions given.

USE OF CALCULATORS IS NOT ALLOWED.

STOP! WAIT FOR THE SIGNAL TO START

PART I (FOR STATISTICS / MATHEMATICS STREAM)

ATTENTION: ANSWER A TOTAL OF SIX [6] QUESTIONS, TAKING AT LEAST TWO [2] FROM EACH GROUP.
(Note: Partial credit may be given for partially correct answer)

GROUP S1
Statistics

1. (a) Consider the following observations.

5, 3, 4, 7, 8, 9, 63, 4, 6, 7, 8, 4, 2, 5, 6, 7, 6, 7, 8, 7, 8, 67, 4, 6, 7.

Suggest a suitable measure of central tendency of the above data with suitable justification. Find out first and second quartile.

(b) Consider the following observations on X and Y.

X	-4	-3	-2	-1	0	1	2	3	4
Y	4	3	2	1	0	1	2	3	4

Find r_{xy}. Does this value of r_{xy} imply any relationship between X and Y for the given data? Explain.

(c) An engineer carried out a simple linear regression analysis with 23 pairs of observations and obtained $r^2 = 0.82$ which is significant at 1% level. Does this prove that there is a linear relationship between the dependent and the independent variables? Would you advise him to carry out any further analysis?

[8+8+4=20]

2. (a) Suppose the population medians of the characteristics of a product manufactured under two different conditions are θ_1 and θ_2 and production persons claim that $\theta_1 < \theta_2$. Suppose two samples of size 7 and 21 respectively were collected to verify the claim.

These two samples were pooled and the median M was computed from the 28 observations. It was observed that M is not equal to any of the sample observations. It was further observed that in the first sample (of size 7) only one value was greater than M, whereas in the second sample (of size 21) 13 values were greater than M. Formulate this as a hypothesis testing problem. Do you think that the claim is valid? Justify.

(b) A large bank sends the details of cheques to be processed to a BPO Company. The cheques are sent sequentially and it is known that the time gap between the arrival of successive cheques follow an exponential distribution. The BPO Company has a software that records the number of cheques arriving every hour. However, the events of no arrivals are not recorded, i.e., records of number of hours when no cheques arrived are not known. Suppose a compiled form of this data for an unknown but long period is made available to you so that the number of hours when j cheques arrived is known for $j > 0$. Explain with required derivations, how you will use this data to estimate the mean inter-arrival time.

[10+10=20]

3. (a) X_1, X_2 and X_3 are i.i.d. with Bernoulli (θ).
Show that $Y = \frac{1}{6}(X_1 + 2X_2 + 3X_3)$ is not a sufficient estimator of the Bernoulli parameter θ.

(b) If x_i ($i = 1, 2, \ldots, n$) are random sample from $N(\mu, \sigma^2)$, show that

$$\frac{\sqrt{\frac{n}{n-1}}(x_1-\overline{x})}{\sqrt{\left[(n-1)s^2-\frac{n}{n-1}(x_1-\overline{x})^2\right]/(n-2)}}, \text{ where } s^2 = \frac{\Sigma(x_i-\bar{x})^2}{n-1}$$

follows t-distribution with $(n-2)$ degrees of freedom.

[10+10=20]

4. An experimenter weighs four solid objects using three weighing machines with the objective of comparing their weights. The weighing machines are located at three different locations. To optimize time, she selects a location, weighs all the four objects sequentially, record respective weights; selects a second location to repeat the process and moves to the last location to complete the process. It is suspected that

the weighing machines may have calibration bias but the measurement variances are equal in all the machines.

What experimental design is most appropriate and why? How should he ensure adherence to the ‘basic principle of randomization’? Write down the statistical model stating necessary assumptions clearly. Write down the normal equations for your model and solve them for estimators of appropriate parameters. Are they unique? Is it possible to estimate the weights of individual objects and why? Is it possible to estimate the difference in weights of two of the objects and why?

[2+2+2+6+2+3+3=20]

5. Suppose it is desired to estimate the total output of a group of N factories such that the sample estimate will lie within 10% of the true value with a confidence coefficient 0.95.

   Determine the sample size n, when N = 500 and the sample is drawn according to SRSWOR method. You may assume that the sample estimate is normally distributed and the population coefficient of variation is 60%.

   [20]

GROUP S2
Probability

6. (a) Show that in odd samples of size n from $U(0, 1)$ population, the mean and variance of the distribution of median are $\frac{1}{2}$ and $\frac{1}{4(n+2)}$ respectively.

   (b) A box contains r red ball and b black balls. Balls are removed at random until the first black ball appears. Show that the expected number of draws is $\frac{(b+r+1)}{(b+1)}$.

[10+10=20]

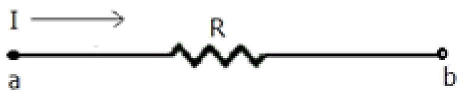
7. (a) Let X and Y be independent random variables where X has a Poison distribution with parameter λ and Y has an exponential distribution with parameter θ. Show that

$$E\left[\left(\frac{Y}{\delta}\right)^X\right] = \left(\frac{\delta\theta}{\delta\theta-\lambda}\right)\, e^{-\lambda},$$

   where, $\delta\theta > \lambda$.

   (b) Consider the simple DC circuit shown in the following figure. The random variables voltage (V) and resistance (R) across the circuit are independently distributed with means μ_V and μ_R and variances σ_V^2 and σ_R^2 . Suppose that the current (I) across the points (a, b) is required to be $\mu_I \pm 3\sigma_I$.

   Prove that $\mu_I \approx \frac{\mu_V}{\mu_R}$ and $\sigma_I^2 \approx \frac{\sigma_V^2 - \mu_I^2\sigma_R^2}{\mu_R^2}$.



[10+10=20]

8. (a) Let $X \sim N(\mu,\sigma^2)$ and $M_X(t)$ be its moment generating function. For a

fixed θ, let Y has the probability density function

$$f_Y(y) = \frac{e^{\theta y} f_X(y)}{M_X(\theta)}, \quad -\infty < y < \infty.$$

Find $Var(Y)$.

(b) There are n boxes of which the r^{th} box contains $(r - 1)$ red balls and $(n - r)$ black balls. You pick one box at random and select two balls at random from the selected box without replacement. Find the probability that second ball is black, given that the first ball is also black.

[13+7=20]

9. (a) Suppose that $X \sim Exp(1)$, $Y \sim U(0,1)$ and are independent.

Let $Z = max\ \{X, Y\}$. Find the expectation of Z.

(b) Consider the random variables X and Y having joint probability density function as

$$f(x,y) = \begin{cases} 3x & if\ 0 < y < x < 1 \\ 0 & elsewhere \end{cases}$$

Determine the value of $V(X - Y)$.

[10+10=20]

10. (a) Let X and Y be distributed in the bivariate normal form with means equal to zero, variances σ_1^2 and σ_2^2 and correlation coefficient ρ. Show that the correlation coefficient between X^2 and Y^2 is ρ^2.

(b) Consider a Poisson Process $\{N(t), t\ \varepsilon\ [0, \infty)\}$ with parameter $\lambda > 0$; where $N(t)$ = total number of occurrences of an event 'E' in an interval of length t. Suppose that each occurrence of 'E' has a constant probability p of being recorded and $(1 - p)$ of being unrecorded $(0 < p < 1)$. Note that the recording of an occurrence is independent of that of the other occurrence.

Let $M(t)$ denote that total number of occurrence of 'E' recorded in an interval of length t.

Show that $\{M(t),\ t\ \varepsilon\ [0, \infty)\}$ is also a Poisson Process with parameter λp.

[10+10=20]

PART II (FOR ENGINEERING STREAM)

ATTENTION: ANSWER A TOTAL OF SIX [6] QUESTIONS TAKING AT LEAST TWO [2] FROM E1. *(Note: Partial credit may be given for partially correct answer)*

GROUP E1
Mathematics

1. (a) Find the number of ordered pairs (x, y) such that $x^2 - 2y^2 = 1$ and x, y are prime numbers.

   (b) Prove that

$$\lim_{x\to 0} \frac{e^x - e^{-x} - 2x}{\tan x - x} = 1$$

   (c) In ΔABC, $\angle B = \angle C$. The points D, E, F are taken on AB, BC and CA such that ΔDEF is an equilateral triangle. Suppose that the measures of the angles $\angle DEB$, $\angle ADF$ and $\angle CFE$ are α, β and γ respectively. Show that $BD : CF = sin\left(\frac{\beta+\gamma}{2}\right) : sin(120^0 - \alpha)$

[6+7+7=20]

2. (a) Let (h, k) be a fixed point where $h,\ k > 0$. A straight line passing through this point cuts the positive direction of the coordinate axes at the points P and Q. Show that the minimum area of the triangle OPQ, O being the origin, is $2hk$ sq unit.

   (b) Consider n distinct real numbers: $a_1, a_2, \ldots, a_n$. A permutation $([1],[2],\ldots,[n])$ of the indices $\{1,2,\ldots,n\}$ is said to be V-shaped if there exists an integer r $(1 \le r \le n)$ such that

$$a_{[1]} \ge a_{[2]} \ge \ldots \ge a_{[r-1]} \ge a_{[r]} \le a_{[r+1]} \le \ldots \le a_{[n]}.$$

   Find the total number of such V-shaped permutations.

(c) Find minimum and maximum values of the function $f(x) = 2x^3 - 15x^2 + 36x$ over the interval [1, 5]. Mention the values of x, where these extrema occur.

[8+7+5=20]

3. (a) Suppose all the integers from 1 to 3333 are listed. Find the number of occurrences of the digit '0' in the list.

(b) Find the values of k ($k > 0$) for which the equation

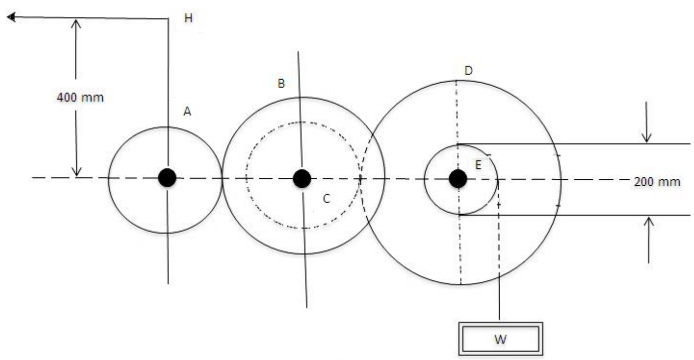
$$|Z - i| + |Z + i| = k$$

represents an ellipse.

[10+10=20]

GROUP E2
Engineering Mechanics

4. (a) A handle H drives a pinion A, which drives a drum E through spur gears B, C and D as shown in the following figure. The length of the handle is 400 mm, and the diameter of the drum is 200 mm. The gear A has 20 teeth, which meshes with gear B of 80 teeth. The gear C has 20 teeth which drives the gear D of 100 teeth. Find the load (W) that can be raised by the drum, if an effort of 10 N is applied at the end of the handle. Consider efficiency of the system as 60%.



(b) The tensions in the two sides of a belt are 1000 N and 800 N respectively. If the speed of the belt is 75 m/s, find the power transmitted by the belt.

(c) Find the necessary difference in tensions in kgf in the two sides of a belt drive when transmitting 20 hp at 30 m/s speed.

[10+5+5=20]

5. (a) A painter's scaffold 10 m long and weighing 0.75 kN is supported in a horizontal position by vertical ropes attached at equal distances from the ends of the scaffold. Find the longest distance from the ends that the ropes may be attached to permit a 1 kN painter to stand safely at one end of the scaffold.

(b) Two solid cubes of equal weight of 100 N are stacked on the floor with one placed over the other. The coefficient of friction between the cubes and that between the lower cube and the floor is 0.3 each. A horizontal force F, applied to the top of the upper cube, is gradually increased. What is the maximum force before the equilibrium is destroyed? How will the equilibrium be destroyed?

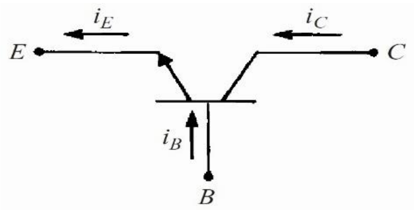
(c) A cantilever beam of negligible weight and length of 5 m carries a uniformly distributed load of 10 kN/m. Draw the SF and BM diagrams.

[5+10+5=20]

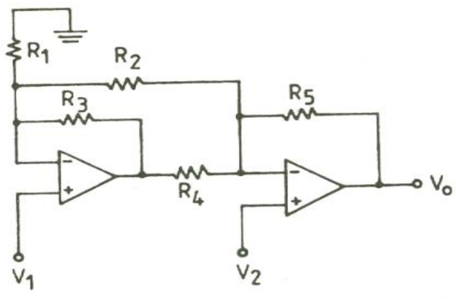
GROUP E3
Electrical & Electronics Engineering

6. (a) Simplify the function $Y = \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$ by using a Karnaugh map and draw the simplified logic circuit using NOR gates only.

(b) In the npn transistor, shown in the figure below, 10^8 holes/μs move from the base to the emitter region while 10^{10} electrons/μs move from the emitter to the base region. An ammeter reads the base current as i_B = 16μA. Determine the emitter current i_E and the collector current i_C. Charge of 1 electron = -1.602 x 10^{-19} C.



(c) Find the differential mode gain, $V_0/(V_1-V_2)$, for the circuit shown below where $R_1 = R_3 = R_4 = R_5$ = 10 kΩ and R_2 = 100 Ω.



[5+6+9=20]

7. (a) A DC shunt machine when run as a motor on no-load takes 440 W at 220 V and runs at 1000 rpm. The field current and armature resistance are 1.0 A and 0.5 Ohm respectively. Calculate the efficiency of the machine when running as a motor taking 40 A from a 220 V supply.

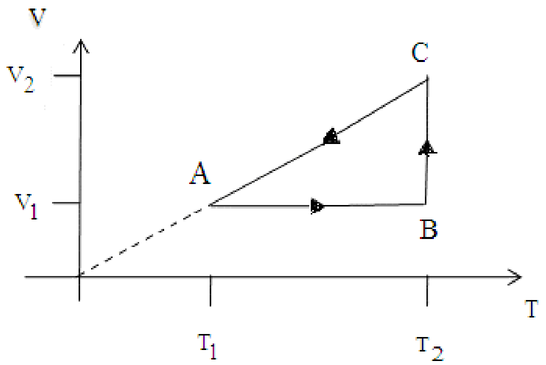
(b) A six-pole induction motor is supplied from a 400 V, three-phase, 50-Hz supply system. The frequency of the rotor induced emf is 2-Hz. Calculate (i) the percentage slip and (ii) the rotor speed.

(c) A 40 KVA transformer has core loss of 250 W and full load copper loss of 750 W. The transformer is used to supply a lighting load with outputs varying as follows - Output: 4-hour at full-load, 8-hour at half-load and the remaining 12-hour at no-load. Calculate the all-day efficiency of the transformer.

[8+5+7=20]

GROUP E4
Thermodynamics

8. (a) The following figure shows a process ABCA performed on an ideal gas of mass *m*. Find the net heat given to the system during the process.



(b) From the first law of thermodynamics show that the change of entropy of a perfect gas of mass *m* is

$$\Delta S = m\left[C_V \ln\left(\frac{T_2}{T_1}\right) + (C_p - C_V)\ln\left(\frac{V_2}{V_1}\right)\right]$$

where V_1 = initial volume of the gas, T_1 = initial temperature of the gas, V_2, T_2 = corresponding values for the final condition, C_p =

specific heat at constant pressure, C_V = specific heat at constant volume.

(c) A sample of 100 g of water is slowly heated from 27^0C to 87^0C.

Calculate the change in the entropy of the water. Specific heat capacity of water = 4200 J kg^{-1} K^{-1}.

[10+6+4=20]

9. (a) A gas of mass 1.5 kg undergoes a quasi-static expansion which follows a relationship $p = a + bV$, where a and b are constants. The initial and final pressures are 1000 kPa and 200 kPa respectively and the corresponding volumes are 0.20 m^3 and 1.20 m^3. The specific internal energy of the gas is given by the relation

$$u = 1.5\,pv - 85 \text{ kJ/kg}$$

where p is in kPa and v is in m^3/kg. Calculate the net heat transfer and the maximum internal energy of the gas attained during expansion.

(b) 1.5 kg of liquid having a constant specific heat of 2.5 kJ/kg K is stirred in a well-insulated chamber causing the temperature to rise by 15^0C. Find ΔE and W for the process.

(c) The properties of a certain fluid are related as follows:

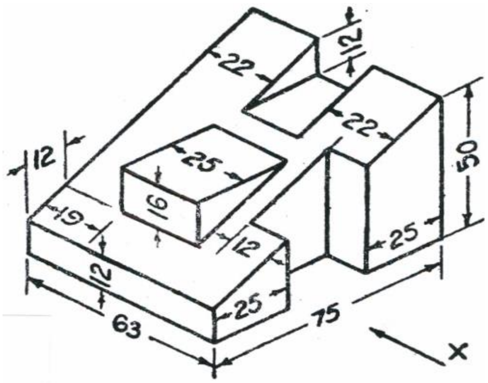
$$u = 196 + 0.718t$$
$$pv = 0.287(t + 273)$$

where u is the specific internal energy (kJ/kg), t is temperature in ^{0}C, p is the pressure (kN/m^2), and v is the specific volume (m^3/kg). For this fluid, find C_V and C_p.

[(5+5)+4+6=20]

GROUP E5
Engineering Drawing

10. (a) Draw or sketch the (i) front view, (ii) side view from the left, and (iii) top view of the given object.



(b) Sketch the top and front views of a double riveted lap joint.

[10+10=20]

11. Two 10 mm thick MS plates are to be joined by using M12×40 hexagonal bolt and nut. One washer is to be used. Draw the three views of the assembly as per standard convention.

[20]