2014

BOOKLET No. **TEST CODE: PQB**

Afternoon

Time: 2 hours

Group	No. of Questions		Maximum Marks
	Total	To be Answered	
Part I (for Statistics/Mathematics Stream)			
S1 (Statistics)	5	A TOTAL OF SIX [6] TAKING AT LEAST TWO [2] FROM EACH GROUP	120
S2 (Probability)	5		
Part II (for Engineering Stream)			
E1 (Mathematics)	3	A TOTAL OF SIX [6] TAKING AT LEAST TWO [2] FROM E1	120
E2 (Engineering Mechanics)	2		
E3 (Electrical and Electronics Engineering)	2		
E4 (Thermodynamics)	2		
E5 (Engineering Drawing)	2		

On the answer-booklet write your Name, Registration Number, Test Code, Number of this Booklet, etc. in the appropriate places.

There are two parts in this booklet as detailed above. Candidates having 'Statistics/Mathematics' background are required to answer questions from Part I only as per instructions given. Those having 'Engineering' background are required to answer questions from Part II only as per instructions given.

USE OF CALCULATOR IS NOT ALLOWED.

STOP! WAIT FOR THE SIGNAL TO START

PART I (FOR STATISTICS / MATHEMATICS STREAM)

ATTENTION: ANSWER A TOTAL OF SIX [6] QUESTIONS, TAKING AT LEAST TWO [2] FROM EACH GROUP. *(Note: Partial credit may be given for partially correct answer)*

GROUP S1
Statistics

1. (a) Drug A is known to be quite effective in curing headache. The performance of drug A in terms of time taken to cure headache has been obtained for 100 randomly chosen patients and the results are given below.

Time to Cure for Drug A (in Minutes)	0 - 5	5 - 10	10 - 15	15 - 20	20 - 25	25 - 30	> 30
Number of Patients	35	24	18	12	5	4	2

Suppose XYZ Pharmaceuticals has come out with drug B and claims that it is superior to drug A. In order to verify the claim of XYZ Pharmaceuticals, data were collected on 100 patients selected randomly and the results are given below.

Time to Cure for Drug B (in Minutes)	0 - 5	5 - 10	10 - 15	15 - 20	20 - 25	25 - 30	> 30
Number of Patients	37	24	19	11	5	3	1

Draw the ogives for the time to cure headache for both the drugs (make free hand sketches on your answer sheet). Observe the ogives closely and offer your comments on the claim made by XYZ Pharmaceuticals about drug B.

(b) Suppose that median software development productivity is typically 23 units or less. A software tool vendor has developed a tool and claims that its usage will improve productivity. In order to check the claim made by the vendor you have used the tool on 10 software development projects and the productivity numbers obtained were 26.4, 24.3, 21.7, 22.4, 24.8, 25.2, 25.7, 26.1, 23.5 and 25.1.

Do you think that the data supports the vendor's claim assuming that you will believe the vendor if the chance of this kind or a worse sample is less than 1% under the assumption of no improvement? Clearly state the hypotheses and the assumptions made by you to carry out the computations.

[10+10=20]

2. Let $x_1, x_2, \ldots, x_n$ be a random sample from the population whose probability density function is given by

$$f(x) = \begin{cases} e^{-(x-\theta)} & if\ x > \theta, \quad -\infty < \theta < \infty \\ 0 & Otherwise \end{cases}.$$

Show that the statistic $x_{(1)} = \min_i x_i$ is a consistent estimator of θ and is sufficient for θ.

[20]

3. Let $\left(y_{1j}; j = 1,2,\ldots n\right), \left(y_{2j}; j = 1,2,\ldots n\right)$ and $\left(y_{3j}; j = 1,2,\ldots n\right)$ be three random samples from $N\left(\mu_1, \sigma^2\right)$, $N\left(\mu_2, \sigma^2\right)$ and $N\left(\mu_3, \sigma^2\right)$ populations respectively, where $\mu_1 = \beta_1$, $\mu_2 = \beta_1 + \beta_2$, and $\mu_3 = \beta_1 + \beta_2 + \beta_3$.

Formulate the above problem as a standard linear model and find the following

a) BLUEs for β_i i = 1, 2, 3, state the necessary results,
b) Error (residual) Sum of Squares, and
c) Dispersion matrix of $\hat{\beta} = \begin{pmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \end{pmatrix}$.

Note: $\begin{pmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$

[20]

4. (a) Let $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$ be a random sample from the discrete distribution with joint probability mass function

$$f_{X,Y}(x,y) = \begin{cases} \frac{\alpha}{4} & for \quad (x,y) = (0,0) \text{ and } (1,1) \\ \frac{2-\alpha}{4} & for \quad (x,y) = (0,1) \text{ and } (1,0) \end{cases}, \text{ where } \quad 0 \leq \alpha \leq 2$$

Find the maximum likelihood estimator of α.

(b) A straight line regression $E(y) = \alpha + \beta x$ is to be fitted using four observations. Assume $Var(y|x) = \sigma^2$ for all x. The values of x at which observations are to be made, lie in the closed interval [−1, 1].

The following choices of the values of x where observations are to be made are available

i) two observations each at $x = -1$ and $x = 1$,
ii) one observation each at $x = -1$ and $x = 1$ and two observations at $x = 0$, and
iii) one observation each at $x = -1, -½, ½, 1$.

If the interest is to estimate the slope with least variance, which of the above data collection plans would you choose and why?

[10+10=20]

5. Suppose a population of size N is divided into L strata and the h^{th} stratum consists of N_h units, $h = 1, 2, \ldots, L$. From the h^{th} stratum an SRSWOR sample of size n_h is drawn, $h = 1, 2, \ldots, L$.

Obtain an unbiased estimator of the population variance based on the stratified random sample.

[20]

GROUP S2
Probability

6. (a) Find the probability of choosing 3 numbers a, b and c from the set of integers $\{1, 2, 3,\ldots,(2k+1)\}$, where k is an positive integer, such that a, b and c are in arithmetic progression.

   (b) One of the sequences of letters XXX, YYY, ZZZ is transmitted over a communication channel with respective probabilities p_1, p_2, p_3, where $(p_1 + p_2 + p_3 = 1)$. The probability that each transmitted letter will be correctly understood is α and the probabilities that the letter will be confused with two other letters are $(1 - \alpha)/2$ and $(1 - \alpha)/2$. It is assumed that the letters are distorted independently.

   i) Find the probability that XXX was transmitted if XYZ was received.
   ii) Find the probability that XYZ was received if XXX was transmitted.

   [10+10=20]

7. (a) Let X and Y be independent standard normal variables. Obtain the moment generating function of XY.

   (b) Draw rough sketch of the density of the random variable Z constructed as follows.

   i) Two observations, say x and y are drawn randomly from $N(5,1)$ and $N(15,1)$ respectively and Z is constructed as $(x + y)/2$.
   ii) An unbiased coin is tossed. If a head turns up then Z is constructed by drawing a random observation from $N(5,1)$, otherwise Z is constructed by drawing a random observation from $N(15,1)$.

   [12+8=20]

8. Determine k so that

$$f(x,y) = \begin{cases} ky \exp(-xy) & if\ 0 \le x < \infty\ and\ 1 \le y \le 4 \\ 0 & Otherwise \end{cases}$$

is a joint probability density function of the two random variables X and Y. Compute their covariance. Derive the conditional probability density function of X given $y = 4$.

[20]

9. Let Z_1, Z_2, Z_3 and Z_4 be four independent standard normal variables. Find the distribution of

$$U = \frac{Z_1 - Z_2 - Z_3 + Z_4}{Z_1 + Z_2 - Z_3 - Z_4}$$

[20]

10. Let $\{X_n \mid n \in \{0, 1, 2, \ldots\}\}$ be a discrete time parameter Markov Chain with state space $I = \{i_0, i_1, i_2, \ldots, i_n\}$ and one step transition matrix $P = ((p_{ij}))_{i,j \in I}$.

Show that $P(X_0 = i_0 \mid X_1 = i_1, X_2 = i_2, \ldots, X_n = i_n) = P(X_0 = i_0 \mid X_1 = i_1)$, $\forall\ n = 0, 1, 2, \ldots, \forall\ \{i_0, i_1, i_2, \ldots, i_n\} \in I$ whenever the left hand side is defined.

[20]

PART II (FOR ENGINEERING STREAM)

ATTENTION: ANSWER A TOTAL OF SIX [6] QUESTIONS TAKING AT LEAST TWO [2] FROM E1. *(Note: Partial credit may be given for partially correct answer)*

GROUP E1
Mathematics

1. (a) Find the number of solutions of $16^{\sin^2 x} + 16^{\cos^2 x} = 10$, $0 \le x \le 2\pi$.

(b) Find the minimum value of the function

$$f(x,y) = \frac{12}{x} + \frac{18}{y} + xy$$

over all positive real numbers x and y. Find also the co-ordinates (x,y) where the function possesses its minimum.

(c) Using the power series $f(x) = \sum_{n=0}^{\infty} \frac{n+1}{n!} x^n$, find the value of

$$\sum_{n=0}^{\infty} \frac{n+1}{n!}$$

[7+8+5=20]

2. (a) If the system of equations

$$\begin{aligned} ax + y + z &= 0 \\ x + by + z &= 0 \qquad (a, b, c \neq 1) \\ x + y + cz &= 0 \end{aligned}$$

has a non-trivial solution, then find the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$.

(b) Find the shortest distance from the origin to the surface given by the equation $xyz = k^3$, where k is a given constant.

(c) Find the area of the region above the x-axis, included between the parabola $y^2 = ax$ and the circle $x^2 + y^2 = 2ax$.

[7+6+7=20]

3. (a) Let $a_1 = 1$ and $a_n = n\,(a_{n-1}+1)$ for $n = 2,3,\ldots$. Define

$$P_n = \left(1+\frac{1}{a_1}\right)\left(1+\frac{1}{a_2}\right)\ldots\left(1+\frac{1}{a_n}\right); n = 1,2,\ldots$$

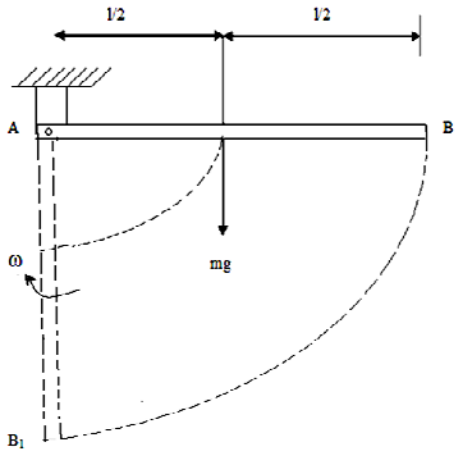
Find $\lim_{n\to\infty} P_n$.

(b) A radioactive substance decays with time such that at any moment the rate of decay of volume is proportional to the volume at that time. The half-life of the substance is the time it takes for half the substance to disappear. Calculate the half-life of the substance if 20% of its volume disappears in 15 years. [Given: ln(2)= 0.6931; ln (5/4) = 0.2231].

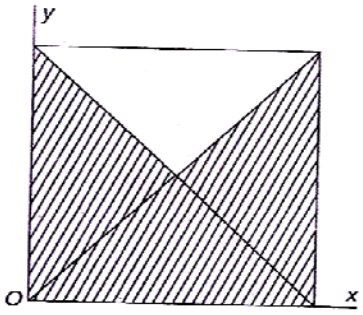
[12+8=20]

GROUP E2
Engineering Mechanics

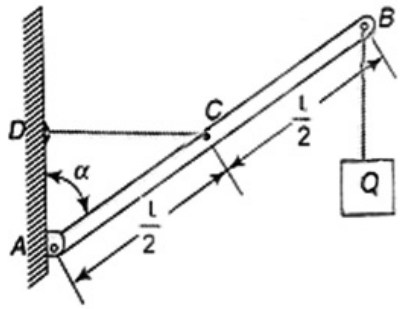
4. (a) If the slender prismatic bar in the following figure is released from rest in the horizontal position AB and allowed to fall under the influence of gravity, show that the angular velocity it will acquire by the time it reaches the vertical position AB_1 is $\varpi = \sqrt{\frac{3g}{l}}$.



(b) Locate the centroid of the shaded three-quarters of the area of a square of dimension *a* as shown in the figure below.



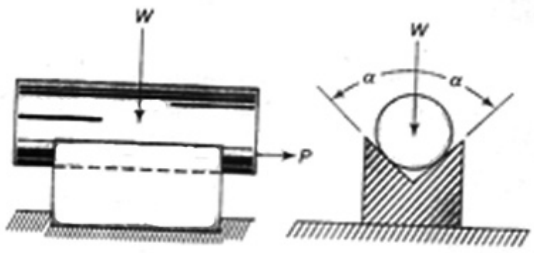
(c) A rigid bar AB of length *l* is supported in a vertical plane and carries a load *Q* at its free end as shown in the figure below. Neglecting the weight of the bar itself, compute the magnitude of the tension induced in the horizontal string CD.



[10+5+5=20]

5. (a) A particle of weight W moves rectilinearly under the action of a force $P\cos\omega t$. Develop the velocity-time and displacement-time equation if at the beginning the displacement and the velocity are zero.

(b) A short right circular cylinder of weight W rests in horizontal 'V' notch having the angle 2α as shown in the figure. If the coefficient of friction is μ, find the horizontal force P necessary to cause slipping to impend.



(c) A beam of rectangular cross-section is to be cut from a circular log of diameter D. Determine the ratio of the depth to the width of the beam to resist maximum bending moment.

[5+5+10=20]

GROUP E3
Electrical & Electronics Engineering

6. (a) Design the circuit for the Boolean function $Y = AB + \overline{A}C$ using NOR gates only.

   (b) Design the circuit for a synchronous counter using JK flip-flops, which will generate the outputs in the order 000, 011, 101, 100, 111, 010, 001, 110 and 000.

   [5+15=20]

7. (a) The induced emf in a dc machine is 200 V at a speed of 1200 rpm. Calculate the electromagnetic torque developed at an armature current of 15 A.

   (b) A 500 kVA, 11000 V/400 V, 50 Hz, single-phase transformer has 100 turns on the secondary winding. Calculate (i) the approximate number of turns in the primary winding, (ii) the approximate value of the primary and the secondary currents and (iii) the approximate maximum value of flux in the core.

   (c) The stator of a three-phase, 8-pole synchronous generator driven at 750 rpm has 72 slots. The winding has been made with 36 coils having 10 turns per coil. Calculate approximately the rms value of the induced emf per phase if the flux per pole is 0.15 Wb, which is sinusoidally distributed. Assume that full-pitch coils have been used and $\sin 10^0 \simeq 0.174$.

   [5 + (2 + 3 + 3) + 7 = 20]

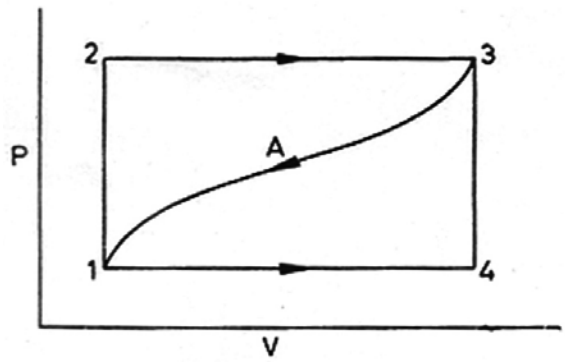
GROUP E4
Thermodynamics

8. (a) A steam engine intakes 100 gram of steam at 100^0C per minute and cools it down to 20^0C. Show that the heat rejected by the steam engine per minute is 62 kcal. The latent heat of vaporization of steam = 540 cal g^{-1}.

(b) A heat engine operates between a cold reservoir at temperature T_2 = 300 K and a hot reservoir at temperature T_1. It takes 200 J of heat from the hot reservoir to deliver 120 J of heat to the cold reservoir in a cycle. What could be the minimum temperature of the hot reservoir?

[10+10=20]

9. (a) When a system changes from state 1 to state 3 along the path 1-2-3 as shown in the figure below, 40 kcal of heat flows in the system, which in turn does 20 kcal of work.



(i) How much work will be done by the system if the heat flowing into the system along the path 1-4-3 is 25 kcal?
(ii) 15 kcal of work is done on the system while returning from state 3 to state 1 along the curved path 3-A-1. How much heat is transferred from or to the system?
(iii) What is the value of the internal energy at state 3 if the internal energy at state 1 is 15 kcal?

(iv) If the value of the internal energy at state 4 is 20 kcal, how much heat will be transferred from or to the system for the paths 1-4 and 4-3? Indicate the direction of flow of heat.

(b) The specific heat of certain gas under constant pressure is related to the temperature by $C_P = 6.55 - \frac{1.5\times10^2}{T} + \frac{1.1\times10^4}{T^2}$ kcal/mole-K, where T is in K. The molecular weight of the gas is 30. Determine the heat transferred per kg of gas if the system is heated from 400 K to 1000 K

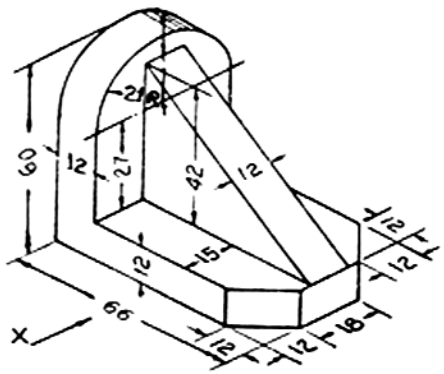
(i) Under constant pressure,
(ii) Under constant volume.

Take R= 1.98 kcal/kg-mol-K.

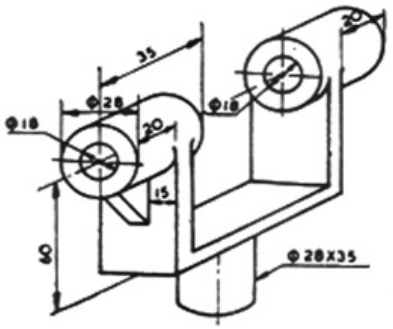
[(2+3+2+3)+(5+5)=20]

GROUP E5
Engineering Drawing

10. (a) Draw the (a) front view, (b) side view from the right, and (c) top view of the given object mentioning dimensions and angle of projection (first angle/ third angle).

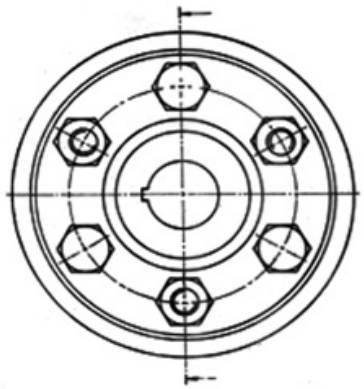


(b) Sketch the two views (top view and front view) of the object shown in the figure. Put the dimensions.



[10+10=20]

11. The front view of the rigid flange shaft coupling is given below. Sketch the sectional side view of the flange shaft coupling. No dimension is to be put.



[20]