2014

Booklet No. TEST CODE: QMB

Afternoon

Questions: 8 **Time: 2 hours**

- On the answer booklet write your Registration number, Test Centre, Test Code and the Number of this Booklet in the appropriate places on the Answer-sheet.

- The test QMB is of short answer type. It has altogether eight questions. A candidate has to answer all questions.

1. a) *Find the value of*

$$\int_0^{\infty} \frac{\beta}{\eta}\left(\frac{x-\mu}{\eta}\right)^{\beta-1} exp\left[-\left(\frac{x-\mu}{\eta}\right)^{\beta}\right] dx,$$

where $\beta>0$, $\eta>0$.

b) *Let* $g(x) = x^6 - x^5 + x^2 - x + 3, -\infty < x < \infty$. *Show that* $g(x) > 0$ *for all* x. (7+8=15)

2. a) *Find the sum of the infinite series*

$$\left(\frac{1}{1.2} + \frac{1.3}{1.2.3.4} + \frac{1.3.5}{1.2.3.4.5.6} + \cdots\right)$$

b) *Show that*

$$1 < \int_0^1 e^{x^2}\, dx < e$$

(7+8=15)

3. a) *Find two positive numbers such that their sum is equal to K and satisfy the property that the product of cube of the first number and square of the second number yields the maximum possible value.*

b) *A cylindrical vessel of volume* $25\frac{1}{7}$ *cubic metre, which is open at the top is to be manufactured from a sheet of metal. Find the dimensions of the vessel, so that the amount of sheet used in manufacturing it, is the least possible.* (7+8=15)

4. a) *If* $a = 11111\cdots1(63\ digits)$,
$b = 1 + 10 + 10^2 + \cdots + 10^6$ *and*
$c = 1 + 10^7 + 10^{14} + \cdots + 10^{56}$, *then show that* $a = bc$.

b) *Suppose a, b and c are distinct non-zero real numbers and the roots of the equation*

$$a(b-c)x^2 + b(c-a)x + c(a-b) = 0$$

are equal, then show that a, b, c are in Harmonic Progression.

(7+8=15)

5. a) *Find the sum of the infinite series*

$$log_9 3 + log_{27} 3 - log_{81} 3 + log_{243} 3 - log_{243} 3 + \cdots$$

b) *Show that the domain of* x *for the real valued function*

$$f(x) = \sqrt{x^{12} - x^9 + x^4 - x + 1}$$

is $-\infty < x < \infty$. (7+8=15)

(Here the square root means the non-negative square root.)

6. a) *Let* $f(x+y) = f(x)\, f(y)$ *and derivatives of f exist at all x and* $y \in R$. *Suppose* $f(5) = 2$ *and* $f'(0) = 3$. *Find the value of* $f'(5)$.

b) *Suppose the velocity V in Km per hour of a motor ship is expressed as function of the cost of fuel consumed per hour, say 'p' rupees per hour as* $V = c.\frac{p}{p+1}$ *where c is constant.*

Also, suppose the fixed operating cost of the ship, other than the fuel cost, is Rupees 'q' per hour of running. Find the velocity of cruising from Port A to Port B, located at a distance of 's' Km from A, so that the cost of cruise is minimum.

$(5 + 10 = 15)$

7. a) *Find the number of 5 digit numbers greater than 40000 that can be formed from the digits 0, 1, 2, 3, 4 and 5 when no repetitions are allowed.*

b) *For a positive integer n, express the function* $g(x) = (3+x)^n$ *as a polynomial:*

$$g(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n.$$

It is given that $\sum_{j=0}^{n} a_j = 4096$.

i) *Find the value of n.*

ii) *Find the largest coefficient* a_j. (5+5+5=15)

8. a) *Let* $0 < p < 1$ *and for m and n integers define*

$$f(m,n) = \begin{cases} (1-p)^n \; if \; 0 \le m \le n < \infty \\ 0 \quad\quad otherwise \end{cases}$$

Then find the value of

$$\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} f(m,n).$$

b) *Let* $\alpha \ge 1$ *and consider the sequence*

$$x_n = \frac{(\alpha+1)^n + \alpha^n + (\alpha-1)^n}{(2\alpha)^n}, \qquad n = 1, 2, 3, \cdots$$

Find $\lim_{n\to\infty} x_n$. (5+10=15)