2016

Booklet No. TEST CODE: QMA

Forenoon

Questions: 30 **Time: 2 hours**

- On the answer booklet write your Name, Registration number, Test Centre, Test Code and the Number of this Booklet in the appropriate places on the Answer-sheet.
- This test has 30 questions. **ANSWER ALL QUESTIONS.** All questions carry equal (4) marks.
- For each of the 30 questions, there are four suggested answers. Only one of the suggested answers is correct. You will have to identify the correct answer to get full credit for that question. Indicate your choice of the correct answer by darkening the appropriate oval completely on the answer-sheet.
- You will get:

4 marks for each **correctly** answered question,
0 marks for each **incorrectly** answered question, and
1 mark for each **unanswered** question.

1. I deposited Rs. X with a bank which was offering 8% interest per year compounded annually for a duration of 5 years. At maturity if I received (approximately) Rs. 264479, X is

   (a) 200000
   (b) 160000
   (c) 180000
   (d) 184479

2. Let n be a positive integer and $a_1, \cdots, a_n$ be any real numbers. Then $n \sum_{i=1}^{n} a_i^2 - (\sum_{i=1}^{n} a_i)^2$ is

   (a) ≥ 0
   (b) ≤ 0
   (c) equal to 0
   (d) equal to $\sum_{i=1}^{n} \sum_{j=1}^{n} a_i a_j$

3. Let $g(x) = \exp(-x)$ for $x > 0$ and
   let $f(x) = \exp(-k)$, for $k - 1 < x \leq k, k = 1, 2, \cdots$.
   Then the area under the curve of $g(x) - f(x)$ is equal to

   (a) $e - 1$
   (b) $e^{-1} - 1$
   (c) $1 - e^{-1}$
   (d) $\frac{e-2}{e-1}$

4. The number of ways to fill each of the four cells of the table with a distinct natural number such that the sum of the numbers is 10 and the sums of the numbers placed diagonally are equal, is

   (a) $2! \times 2!$
   (b) $4!$
   (c) $2(4!)$
   (d) $2(2! \times 2!)$

5. If $f(x) = \frac{1}{1-x}, x \neq 0, 1,$then the graph of the function $y = f\{f(f(x)\}, x > 1$ is

   (a) A circle
   (b) An ellipse
   (c) A straight line
   (d) A pair of straight lines

6. A particle, initially at origin, moves along x-axis according to the rule $\frac{dx}{dt} = x + 4$. The time taken by the particle to traverse a distance of 96 units is

   (a) $\log_5 e$
   (b) $2 \log_e 5$
   (c) $2 \log_5 e$
   (d) $\frac{\log_5 e}{2}$

7. If the value of the determinant $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}$ is positive then

   (a) $abc > 1$
   (b) $abc > -8$
   (c) $abc < -8$
   (d) $abc > -2$

8. If $\log_2(5.2^x + 1), \log_4(2^{1-x} + 1)$ and 1 are in A.P., then x equals to

   (a) $\log_2 5$
   (b) $1 - \log_2 5$
   (c) $\log_5 2$
   (d) none of the above

9. Let $S \subset (0, \pi)$ denote the set of the values of x satisfying the equation

$$8^{\sum_{k=1}^{\infty} |\cos(x)|^{k-1}} = 4^3$$

   then $S =$

   (a) $\{\frac{\pi}{3}\}$
   (b) $\{\frac{\pi}{3}, -\frac{2\pi}{3}\}$
   (c) $\{-\frac{\pi}{3}, \frac{2\pi}{3}\}$
   (d) $\{\frac{\pi}{3}, \frac{2\pi}{3}\}$

10. If $|a| < 1$ and $|b| < 1$ then the sum of the infinite series

$$1 + (1 + a)b + (1 + a + a^2)b^2 + (1 + a + a^2 + a^3)b^3 + \cdots$$

is

(a) $\frac{1}{(1-a)(1-b)}$

(b) $\frac{1}{(1-a)(1-ab)}$

(c) $\frac{1}{(1-b)(1-ab)}$

(d) $\frac{1}{(1-a)(1-b)(1-ab)}$

11. Let

$$f(x) = \begin{cases} \frac{x-4}{|x-4|} + a & \text{for } x < 4 \\ a + b & \text{for } x = 4 \\ \frac{x-4}{|x-4|} + b & \text{for } x > 4 \end{cases}$$

Then $f(x)$ is continuous at $x = 4$ when

(a) $a = 0, b = 0$

(b) $a = 1, b = 1$

(c) $a = -1, b = 0$

(d) $a = 1, b = -1$

12. Let $f(x) = \frac{x-2}{4}$ and $g(x) = 2x^2 + 4$, then $g(f(x))$ is

(a) $\frac{x^2}{8} - 0.5x + 4.5$

(b) $\frac{(x^2-x)}{8} + 3.5$

(c) $\frac{(x-2)^2}{4} + 1$

(d) $2(x - 2)^2 + 4$

13. Suppose $f(x)$ is differentiable at $x = 1$ and

$$\lim_{h \to 0} \frac{f(1 + h)}{h} = 5,$$

then $f(1)$ equals

(a) 0

(b) 6

(c) 3

(d) 4

14. Two fair dice are rolled one after the other. The probability that the number on the top of the first die is smaller than that of the second one is equal to

(a) $\frac{7}{18}$
(b) $\frac{1}{2}$
(c) $\frac{5}{12}$
(d) $\frac{3}{4}$

15. The number of all possible positive integral solutions of the equation $xyz = 30$ is

(a) 27
(b) 25
(c) 26
(d) none of the above

16. If $D = \begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$, then $\frac{D}{(n!)^3} - 4$ is

(a) 0
(b) a multiple of n
(c) a multiple of $n+1$
(d) none of the above

17. If $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ then

(a) $a = \cos 2\theta, b = \sin 2\theta$
(b) $a = 1, b = 1$
(c) $a = \sin 2\theta, b = \cos 2\theta$
(d) none of the above

18. If $\overrightarrow{a} + \overrightarrow{b}$ is at right angles to $\overrightarrow{b}$ and $2\overrightarrow{b} + \overrightarrow{a}$ is at right angles to $\overrightarrow{a}$ then

(a) $a = \sqrt{2}b$
(b) $a = 2b$
(c) $a = b$
(d) $2a = b$

19. $\int(1+x-x^{-1})e^{x+x^{-1}}dx =$

   (a) $(x+1)e^{x+x^{-1}}+c$
   (b) $(x-1)e^{x+x^{-1}}+c$
   (c) $-xe^{x+x^{-1}}+c$
   (d) $xe^{x+x^{-1}}+c$

20. If $a_1, a_2, ..., a_n$ are in harmonic progression, then

$$\frac{a_1}{a_2+a_3+a_4+...a_n}, \frac{a_2}{a_1+a_3+a_4+...a_n}, \cdots, \frac{a_n}{a_1+a_2+a_3+a_4+...a_{n-1}}$$

   are in

   (a) arithmetic Progression
   (b) geometric Progression
   (c) harmonic Progression
   (d) none of the above

21. If f is a real valued differentiable function satisfying

$$|f(x)-f(y)| \leq (x-y)^2,$$

   for all real numbers x, y and $f(0)=0$, then $f(1)$ equals to

   (a) 2
   (b) 1
   (c) -1
   (d) 0

22. If $\log_2 x + \log_x 2 = \frac{10}{3} = \log_2 y + \log_y 2$ and $x \neq y$, then $x+y$ is equal to

   (a) 2
   (b) $\frac{65}{8}$
   (c) $\frac{37}{6}$
   (d) none of the above

23. Let $A = \begin{bmatrix} a & -1 & 4 \\ 0 & b & 7 \\ 0 & 0 & 3 \end{bmatrix}$ be a matrix with real entries. If the sum and product of all the eigen values of A are 10 and 30 respectively, then $a^2 + b^2$ is

(a) 29
(b) 30
(c) 58
(d) 60

24. If $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$, then $I + 2A + 3A^2 + 4A^3 + ...$ is equal to

(a) $\begin{bmatrix} 4 & 1 \\ -4 & 0 \end{bmatrix}$
(b) $\begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$
(c) $\begin{bmatrix} 5 & 2 \\ -8 & -3 \end{bmatrix}$
(d) $\begin{bmatrix} 5 & 2 \\ -3 & -8 \end{bmatrix}$

25. The domain of $y = \frac{1}{\sqrt{|x|-x}}$, where y is well defined

(a) $[0, \infty)$
(b) $(-\infty, 0)$
(c) $(0, \infty)$
(d) $[1, \infty)$

26. If the product of n positive numbers is 1, their sum is

(a) a positive integer
(b) divisible by n
(c) equals to $n + \frac{1}{n}$
(d) never less than n

27. The value of

$$\sum_{i=0}^{10} \sum_{j=i}^{10} {}^{10}C_j \; {}^{j}C_i$$

is equal to

(a) $2^{10} - 1$
(b) 2^{10}
(c) $3^{10} - 1$
(d) 3^{10}

28. Let A and B be two events such that $P(A) = 0.3$ and $P(A \cup B) = 0.8$. If A and B are independent events, then P(B) is

(a) $\frac{2}{7}$
(b) $\frac{3}{7}$
(c) $\frac{4}{7}$
(d) $\frac{5}{7}$

29. For any integer $n \geq 1$, let $S_n = \sum_{r=1}^{n} t_r = \frac{1}{6}n(2n^2 + 9n + 13)$. Then $\sum_{r=1}^{n} \sqrt{t_r}$ equals to

(a) $\frac{1}{2}n(n+1)$
(b) $\frac{1}{2}n(n+2)$
(c) $\frac{1}{2}n(n+3)$
(d) $\frac{1}{2}n(n+4)$

30. Out of 15 tickets marked with numbers from 1 to 15, three are drawn at random. What is the probability that the numbers on them are in arithmetic progression?

(a) $\frac{9}{39}$
(b) $\frac{9}{15}$
(c) $\frac{13}{261}$
(d) none of the above