2014

Booklet No. TEST CODE: QMA

Forenoon

Questions: 30 **Time: 2 hours**

- On the answer booklet write your Name, Registration number, Test Centre, Test Code and the Number of this Booklet in the appropriate places on the Answer-sheet.

- This test has 30 questions. **ANSWER ALL QUESTIONS.** All questions carry equal (4) marks.

- For each of the 30 questions, there are four suggested answers. Only one of the suggested answers is correct. You will have to identify the correct answer to get full credit for that question. Indicate your choice of the correct answer by darkening the appropriate oval completely on the answer-sheet.

- You will get:

  4 marks for each **correctly** answered question,
  0 marks for each **incorrectly** answered question, and
  1 mark for each **unanswered** question.

1. *If* $s = a + b + c$, *then the value of the determinant*

$$\begin{vmatrix} s+c & a & b \\ c & s+a & b \\ c & a & s+b \end{vmatrix}$$

is

(A) $2s^2$ (B) $2s^3$ (C) s^3 (D) $3s^3$

2. *If* $A = \begin{bmatrix} \alpha & 0 \\ 5 & 1 \end{bmatrix}$ *and* $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, *whenever* $A^2 = B$, *the value of* α *is*

(A) 5 (B) -1 (C) 11 (D) none of the above

3. *Two vectors* $\vec{a}$ *and* $\vec{b}$ *are non-zero and non-collinear. The value of* x *for which the vectors* $\vec{p} = (x-2)\vec{a} + \vec{b}$ *and* $\vec{q} = (x+1)\vec{a} - \vec{b}$ *are collinear is equal to*

(A) 2/3 (B) 2 (C) 1 (D) 1/2

4. *In an examination of 9 papers, a candidate has to pass in more papers than the number of papers in which he fails in order to be successful. Find how many ways he can be successful.*

(A) 256 (B) 216 (C) 220 (D) 272

5. *The rank of the matrix* $\begin{bmatrix} 2 & 3 & 4 \\ 5 & 7 & 1 \\ 8 & 9 & 6 \end{bmatrix}$ *is*

(A) 2 (B) 3 (C) 1 (D) 0

6. *The value of the determinant*

$$\left| \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 1 \\ 2 & 2 & 3 \end{bmatrix} \right|$$

is

(A) 25 (B) 9 (C) 15 (D) None of these

7. *Expand each side of a cube by 50%. Then the surface area will expand by*

(A) 50% (B) 300% (C) 150% (D) 125%

8. *Find the maximum value of* $f(x) = \frac{x}{4+x+x^2}$ *on* $[-1, 1]$.

(A) $-\frac{1}{4}$ (B) 0 (C) $\frac{1}{2}$ (D) $\frac{1}{6}$

9. *The number of integral points (both coordinates are integers) that lie exactly in the interior of a triangle with vertices* $(0,0)$, $(0,21)$ *and* $(21,0)$ *is*

(A) 133 (B) 190 (C) 233 (D) 105

10. *Let the matrix* $D = diag(d_1, d_2, d_3, \cdots, d_n)$, *where* $d_i \neq 0$ *for* $i = 1, 2, 3, \cdots, n$ *then* D^{-1} *is equal to*

(A) D (B) $2D$

(C) $diag({d_1}^{-1}, {d_2}^{-1}, {d_3}^{-1}, \cdots, {d_n}^{-1})$ (D) $Adj\ D$

11. *A ball is dropped from a height of 10 metres. On each bounce the ball bounces to three quarters of the previous height attained. The distance travelled by the ball before it comes to rest, assuming that it does not move sideways, is*

(A) 60 meters (B) 70 meters

(C) 80 meters (D) 90 meters

12. *Evaluate* $\lim_{x \to 0} \frac{x - tanx}{x^3}$

(A) 1/3 (B) 2/3 (C) 1 (D) $-1/3$

13. *For $x \neq 0$, consider the function*

$$f(x) = \frac{1}{x} - \frac{2}{\exp(2x) - 1}$$

then $\lim_{x \to 0} f(x)$ is

(A) not defined (B) 0 (C) 1 (D) 2/3

14. *A class has 15 students. In a test with 100 questions, each question carrying 1 mark, the average score was 80; no negative or fractional marks were given on any of the questions. If the maximum score obtained by any of the students was 85, the minimum possible value for the lowest score obtained was*

(A) 10 (B) 12 (C) 15 (D) 75

15. *If*

$$f(x,y) = \begin{cases} \exp(-y) & if\ 0 < x < y < \infty \\ 0 & otherwise, \end{cases}$$

then

$$\int_0^\infty \int_0^\infty f(x,y)\, dx\, dy$$

is

(A) 2 (B) 1 (C) $\exp(-2)$ (D) none of the above

16. *If $f(x+y) = f(x)f(y)\ \forall\ x, y \in R$ and $f(0) \neq 0$, then the function*

$$g(x) = \frac{f(x)}{1 + \{f(x)\}^2}$$

is

(A) an even function (B) an odd function

(C) an odd function if $f(x) > 0$ (D) neither even nor odd

17. *If $e^{e^x} = a_0 + a_1 x + a_2 x^2 + \cdots$, a power series, then*

(A) $a_0 = 1$ (B) $a_0 = e$ (C) $a_0 = e^e$ (D) $a_0 = e^2$

18. *If* $A = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$, *then* $A^4 =$

(A) 27A (B) 81A (C) 243A (D) 729A

19. *If* $log_{10}(x^3 + y^3) - log_{10}(x^2 + y^2 - xy) \leq 2$, *and* $x > 0, y > 0$, *then the maximum value of* xy *is*

(A) 2500 (B) 3000 (C) 1200 (D) 3500

20. *The value of the integral*

$$\int \frac{(x+1)e^x}{(x+2)^2}\,dx$$

is

(A) $\frac{e^{2x}}{x+2} + C$ (B) $\frac{e^x}{(x+2)^2} + C$ (C) $\frac{e^x}{x+2} + C$ (D) $\frac{e^{2x}}{(x+2)^2} + C$

21. *A parallelogram is cut by 'm' lines parallel to both the sides. Determine the number of parallelograms that can be formed.*

(A) $(m+2)^2$ (B) m^2 (C) $\binom{m}{2} \times \binom{m}{2}$ (D) $\binom{m+2}{2} \times \binom{m+2}{2}$

22. $\lim_{x \to 0} \frac{sin(\pi\, cos^2 x)}{x^2}$ *equals to*

(A) $-\pi$ (B) π (C) $\frac{\pi}{2}$ (D) 1

23. *If* $f(x) = [x]$ and $g(x) = x - [x]\ \forall\ x \in R$, *where* $[x]$ = *greatest integer less than or equal to x, then which of the following function is the zero function?*

(A) $f(x) + g(x)$ (B) $f(x)g(x)$
(C) $f(x) - g(x)$ (D) $f(g(x))$

24. *If* $x > 1,\ y > 1,\ z > 1$ *are in G.P., then*

$$\frac{1}{1 + log\, x}, \quad \frac{1}{1 + log\, y}, \quad \frac{1}{1 + log\, z}$$

are in

(A) A. P. (B) H. P. (C) G. P. (D) None of the above

25. *Two numbers are selected from the set* $\{1, 2, 3, \cdots, 3n\}$. *The number of ways in which this can done, if the sum of the selected numbers is divisible by 3, is equal to*

(A) $\frac{3n(3n-1)}{2}$ (B) $\frac{n(3n-1)}{2}$ (C) $\frac{3n(n-1)}{2}$ (D) $\frac{n(n-1)}{2}$

26. *The value of* $\int_a^b \frac{|x|}{x} dx$ *when* $(a < b < 0$ *or* $b > a > 0)$ *is*

(A) $b - a$ (B) $a - b$ (C) $a + b$ (D) $|b| - |a|$

27. *The value of*

$$\sum_{n=1}^{10} \int_{-2n-1}^{-2n} sin^{27} x \, dx + \sum_{n=1}^{10} \int_{2n}^{2n+1} sin^{27} x \, dx$$

is equal to

(A) 27 (B) 54 (C) -54 (D) 0

28. *The value of the integral* $\int_0^1 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{1-x}} dx$ *is*

(A) -1 (B) 2 (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$

29. *Let* $f(x) = \frac{x-[x]}{1+x-[x]}$, $x \in R$ *and* $[x]$ = *greatest integer less than or equal to* x. *The range of* f *is*

(A) $[0, 1]$ (B) $[0, \frac{1}{2}]$ (C) $[0, \frac{1}{2})$ (D) $(0,1)$

30. *If* $x = a(\theta - sin\,\theta); y = a(1 - cos\,\theta)$, *where* $a \neq 0$ *and* $0 < \theta < \pi$, *the value of* $\frac{d^2y}{dx^2}$ *is*

(A) $\frac{a}{y}$ (B) $\frac{a^2}{y}$ (C) $-\frac{a^2}{y}$ (D) $-\frac{a}{y^2}$