**2014**

Booklet No. TEST CODE: PEA

*Forenoon*

**Questions: 30** **Time: 2 hours**

- On the answer booklet write your Name, Registration number, Test Centre, Test Code and the Number of this Booklet in the appropriate places on the Answer-sheet.

- This test has 30 questions. **ANSWER ALL QUESTIONS.** All questions carry equal (4) marks.

- For each of the 30 questions, there are four suggested answers. Only one of the suggested answers is correct. You will have to identify the correct answer to get full credit for that question. Indicate your choice of the correct answer by darkening the appropriate oval completely on the answer-sheet.

- You will get:

  4 marks for each **correctly** answered question,
  0 marks for each **incorrectly** answered question, and
  1 mark for each **unanswered** question.

1. $\lim\limits_{x \to 0+} \dfrac{\sin\{\sqrt{x}\}}{\{\sqrt{x}\}}$, where $\{x\}$ = decimal part of *x*, is

   (a) 0 (b) 1 (c) non-existent (d) none of these

2. $f : [0,\ 1] \to [0,\ 1]$ is continuous. Then it is true that

   (a) $f(0) = 0,\ \ f(1) = 1$
   (b) f is differentiable only at $x = \dfrac{1}{2}$
   (c) $f'(x)$ is constant for all $x \in (0,1)$
   (d) $f(x) = x$ for at least one $x \in [0,1]$

3. $f(x) = |x - 2| + |x - 4|$. Then *f* is
   (a) continuously differentiable at x = 2
   (b) differentiable but not continuously differentiable at $x = 2$
   (c) *f* has both left and right derivatives at $x = 2$
   (d) none of these

4. In an examination of 100 students, 70 passed in Mathematics, 65 passed in Physics and 55 passed in Chemistry. Out of these students, 35 passed in all the three subjects, 50 passed in Mathematics and Physics, 45 passed in Mathematics and Chemistry and 40 passed in Physics and Chemistry. Then the number of students who passed in exactly one subject is

   (a) 30 (b) 25 (c) 10 (d) none of these

5. The square matrix of the matrix $\begin{vmatrix} a & b \\ c & 0 \end{vmatrix}$ is a null matrix if and only if
   (a) $a = b = c = 0$
   (b) $a = c = 0$, $b$ is any non-zero real number
   (c) $a = b = 0$, $c$ is any non-zero real number
   (d) $a = 0$ and either $b = 0$ $or$ $c = 0$

6. If the positive numbers $x$, $y$, $z$ are in harmonic progression, then $\log(x + z) + \log(x - 2y + z)$ equals

(a) $4\log(x-z)$ (b) $3\log(x-z)$
(c) $2\log(x-z)$ (d) $\log(x-z)$

7. If $f$(x+2y, x-2y) = xy, then $f$(x, y) equals

(a) $\frac{x^2-y^2}{8}$ (b) $\frac{x^2-y^2}{4}$ (c) $\frac{x^2+y^2}{4}$ (d) none of these

8. The domain of the function $f(x) = \sqrt{x^2-1} - \log\left(\sqrt{1-x}\right)$, $x \geq 0$, is

(a) $(-\infty, -1)$ (b) $(-1, 0)$ (c) null set (d) none of these

9. The graph of the function $y = \log(1-2x+x^2)$ intersects the x axis at

(a) 0, 2 (b) 0, - 2 (c) 2 (d) 0

10. The sum of two positive integers is 100. The minimum value of the sum of their reciprocals is

(a) $\frac{3}{25}$ (b) $\frac{6}{25}$ (c) $\frac{1}{25}$ (d) none of these

11. The range of the function $f(x) = 4^x + 2^x + 4^{-x} + 2^{-x} + 3$, where $x \in (-\infty, \infty)$, is

(a) $\left(\frac{3}{4}, \infty\right)$ (b) $\left[\frac{3}{4}, \infty\right)$ (c) $(7, \infty)$ (d) $[7, \infty)$

12. The function $f: R \to R$ satisfies $f(x+y) = f(x) + f(y) \, \forall \, x, y \in R$, where $R$ is the real line, and $f(1) = 7$. Then $\sum_{r=1}^{n} f(r)$ equals

(a) $\frac{7n}{2}$ (b) $\frac{7(n+1)}{2}$ (c) $\frac{7n(n+1)}{2}$ (d) $7n(n+1)$

13. Let $f$ and $g$ be differentiable functions for $0 < x < 1$ and $f(0) = g(0) = 0, f(1) = 6$. Suppose that for all $x \in (0, 1)$, the equality $f'(x) = 2g'(x)$ holds. Then $g(1)$ equals

(a) 1 (b) 3 (c) -2 (d) - 1

14. Consider the real valued function $f(x) = ax^2 + bx + c$ defined on [1, 2]. Then it is always possible to get a $k \in (1, 2)$ such that

(a) $k = 2a + b$ (b) $k = a + 2b$ (c) $k = 3a + b$ (d) none of these

15. In a sequence the first term is $\frac{1}{3}$. The second term equals the first term divided by 1 more than the first term. The third term equals the second term divided by 1 more than the second term, and so on. Then the 500th term is

(a) $\frac{1}{503}$ (b) $\frac{1}{501}$ (c) $\frac{1}{502}$ (d) none of these

16. In how many ways can three persons, each throwing a single die once, make a score of 10?

(a) 6 (b) 18 (c) 27 (d) 36

17. Let $a$ be a positive integer greater than 2. The number of values of $x$ for which $\int_a^x (x + y)dy = 0$ holds is:

(a) 1 (b) 2 (c) a (d) a + 1

18. Let $(x^*, y^*)$ be a solution to any optimization problem $max_{(x,y)\in\Re^2} f(x, y)$ subject to $g_1(x, y) \leq c_1$. Let $(x', y')$ be a solution to the same optimization problem $max_{(x,y)\in\Re^2} f(x, y)$ subject to $g_1(x, y) \leq c_1$ with an added constraint that $g_2(x, y) \leq c_2$. Then which one of the following statements is always true?

(a) $f(x^*, y^*) \geq f(x', y')$ (b) $f(x^*, y^*) \leq f(x', y')$
(c) $|f(x^*, y^*)| \geq |f(x', y')|$ (d) $|f(x^*, y^*)| \leq |f(x', y')|$

19. Let $(x^*, y^*)$ be a real solution to: $max_{(x,y)\in\Re^2}\sqrt{x} + y$ subject to $px + y \leq m$, where $m > 0, p > 0$ and $y^* \in (0, m)$. Then which one of the following statements is true?
    (a) $x^*$ depends only on $p$ (b) $x^*$ depends only on $m$
    (c) $x^*$ depends on both $p$ and $m$ (d) $x^*$ is independent of both $p$ and $m$.

20. Let $0 < a_1 < a_2 < 1$ and let $f(x; a_1, a_2) = -|x - a_1| - |x - a_2|$. Let X be the set of all values of x for which $f(x; a_1, a_2)$ achieves its maximum. Then
    (a) $X = \{x | x \in \left\{\frac{a_1}{2}, \frac{1+a_2}{2}\right\}\}$ (b) $X = \{x | x \in \{a_1, a_2\}\}$
    (c) $X = \{x | x \in \left\{0, \frac{a_1+a_2}{2}, 1\right\}\}$ (d) $X = \{x | x \in [a_1, a_2]\}$.

21. Let A and B be two events with positive probability each, defined on the same sample space. Find the correct answer:
    (a) P(A/B) > P(A) always (b) P(A/B) < P(A) always
    (c) P(A/B) > P(B) always (d) None of the above

22. Let A and B be two mutually exclusive events with positive probability each, defined on the same sample space. Find the correct answer:
    (a) A and B are necessarily independent
    (b) A and B are necessarily dependent
    (c) A and B are necessarily equally likely
    (d) None of the above

23. The salaries of 16 players of a football club are given below (units are in thousands of rupees).
    100, 100, 111, 114, 165, 210, 225, 225, 230,
    575, 1200, 1900, 2100, 2100, 2650, 3300
    Now suppose each player received an extra Rs. 200,000 as bonus. Find the correct answer:
    (a) Mean will increase by Rs. 200,000 but the median will not change
    (b) Both mean and median will be increased by Rs. 200,000
    (c) Mean and standard deviation will both be changed
    (d) Standard deviation will be increased but the median will be unchanged

24. Let Pr(X = 2) = 1. Define $\mu_{2n} = E(X-\mu)^{2n}, \mu = E(X)$. Then:

(a) $\mu_{2n}$= 2 (b) $\mu_{2n}$= 0 (c) $\mu_{2n} > 0$ (d) None of the above

25. Consider a positively skewed distribution. Find the correct answer on the position of the mean and the median:
(a) Mean is greater than median (b) Mean is smaller than median
(c) Mean and median are same (d) None of the above

26. Puja and Priya play a fair game (i.e. winning probability is ½ for both players) repeatedly for one rupee per game. If originally Puja has *a* rupees and Priya has *b* rupees (where a > b), what is Puja's chances of winning all of Priya's money, assuming the play goes on until one person has lost all her money?

(a) 1 (b) 0 (c) b/(a+b) (d) a/(a+b)

27. An urn contains *w* white balls and *b* black balls (*w* > *0*) and (*b* > *0*). The balls are thoroughly mixed and two are drawn, one after the other, *without* replacement. Let *Wi* denote the outcome 'white on the *i*-th draw' for *i* = 1, 2. Which one of the following is true?
(a) P(W2) = P(W1) = w/(w+b)
(b) P(W2) = P(W1) = (w-1)/(w+b-1)
(c) P(W1) = w/(w+b), P(W2) = (w-1)/(w+b-1)
(d) P(W1) = w/(w+b), P(W2) = {w(w-1)}/{(w-b)(w+b-1)}

28. A bag contains four pieces of paper, each labeled with one of the digits 1, 2, 3, 4, with no repeats. Three of these pieces are drawn, one at a time without replacement, to construct a three-digit number. What is the probability that the three-digit number is a multiple of 3?

(a) ¾ (b) ½ (c) ¼ (d) 9/24

29. Consider two random variables X and Y where X takes values -2,-1,0,1,2 each with probability 1/5 and Y=|X|. Which of the following is true?
    (a) The variables X and Y are independent and Pearson's correlation coefficient between X and Y is 0.
    (b) The variables X and Y are dependent and Pearson's correlation coefficient between X and Y is 0.
    (c) The variables X and Y are independent and Pearson's correlation coefficient between X and Y is 1.
    (d) The variables X and Y are dependent and Pearson's correlation coefficient between X and Y is 1.

30. Two friends who take the metro to their jobs from the same station arrive to the station uniformly randomly between 7 and 7:20 in the morning. They are willing to wait for one another for 5 minutes, after which they take a train whether together or alone. What is the probability of their meeting at the station?

    (a) 5/20 (b) 25/400 (c) 10/20 (d) 7/16